

Math 1010 - Lecture 8 Notes

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In our everyday lives when we say that in order to be President of the United States a person must be at least 35 years old, in order to get a senior discount a person must be at least 65 years old, that when you drive on the freeway you must drive no more than 65 miles per hour, or that you must be at least four feet tall to ride the roller coaster, we're using a mathematical concept called an *inequality*. Inequalities turn up all over the place, in life as well as in math.

1 Algebraic Inequalities

An algebraic inequality is an inequality that contains one or more more variable term. For example:

$$4x + 5 < 9$$

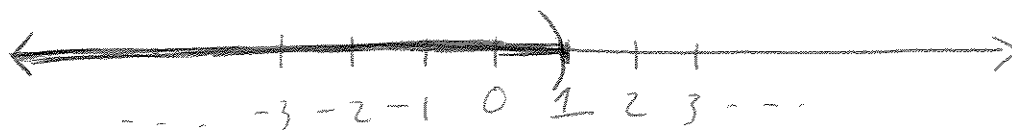
would be an algebraic inequality. Here, the set of values of x that *satisfies* the inequality are those values of x for which the inequality is true. In this case, the inequality will be true if x is less than 1, and false if x is greater than or equal to 1.

Note that there are usually quite a few values that satisfy an inequality. In our example the set of values that satisfy the inequality is *every* single number less than 1. On the other hand, if we were given the algebraic equality:

$$4x + 5 = 9$$

the set of values that satisfy this equality would be only one number, namely 1.

Now, we can graph the values that satisfy an inequality on the number line. In our example of $4x + 5 < 9$, the graph would look like:

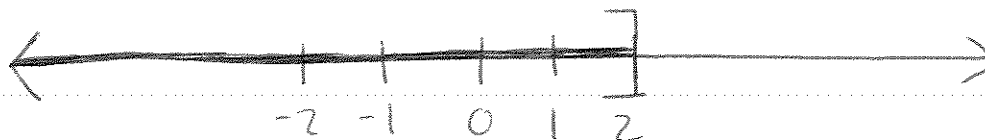


$$x < 1.$$

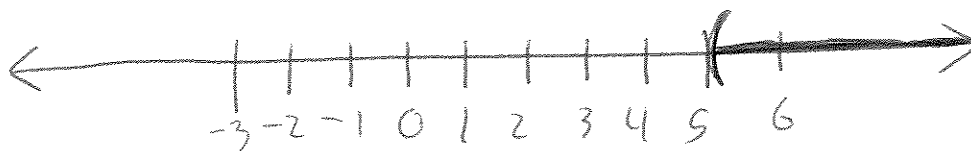
Examples:

Graph the values of x that satisfy the following inequalities on the number line:

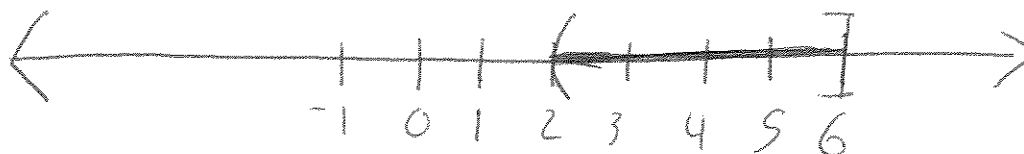
1. $x \leq 2$



2. $x > 5$



3. $2 < x \leq 6$



Note that in the last situation we had what's known as a *compound inequality*. Also, when our inequality includes an endpoint, we mark that with a bracket [or], while if it does not we mark that with parentheses (or).

2 Solving Inequalities

Up until now we've skipped over how we actually calculated that $4x + 5 < 9$ was satisfied for $x < 1$, so let's go over that in detail. Here's how the calculation would go down:

$$4x + 5 < 9.$$

Subtract 5 from both sides to get:

$$4x < 4.$$

Divide both sides by 4 to get:

$$x < 1.$$

And we're done.

Now, what we've seen here is that, when solving an algebraic inequality we can use almost all the rules we've learned about solving algebraic equalities. Our goal is to get x on one side and numbers on the other, and to do this we can add or subtract from both sides, and we can multiply or divide both sides by a positive number. *However*, if we multiply or divide both sides by a negative number we have to be careful.

We can take a simple example:

$$3 < 5.$$

If we multiply both sides of this by -1 the left side becomes -3 and the right side becomes -5 , and the proper inequality would then be:

$$-3 > -5.$$

We note here that our inequality switched its direction. This is a general rule. Namely, if we multiply or divide both sides of an algebraic inequality by a negative number, we have to switch the direction of the inequality.

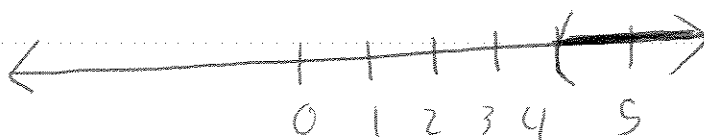
Let's do some examples.

3 Examples

Solve the following inequalities, and sketch the solution on the real number line.

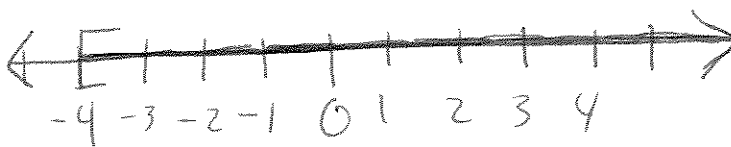
1. $z - 4 > 0$

$$\begin{array}{r} z - 4 > 0 \\ + 4 \quad + 4 \\ \hline \Rightarrow z > 4 \end{array}$$



2. $-6x \leq 24$

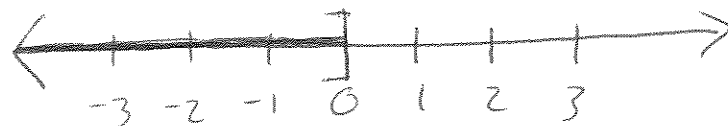
$$\begin{array}{r} -6x \leq 24 \\ \hline -6 \quad -6 \\ \hline x \geq -4 \end{array}$$



$$3. 2(4 - z) \geq 8(1 + z)$$

$$\begin{aligned} 2(4) - 2(z) &\geq 8(1) + 8z \\ \Rightarrow 8 - 2z &\geq 8 + 8z \\ \begin{array}{r} -8 \qquad -8 \\ \hline \end{array} \\ \Rightarrow -2z &\geq 8z \end{aligned}$$

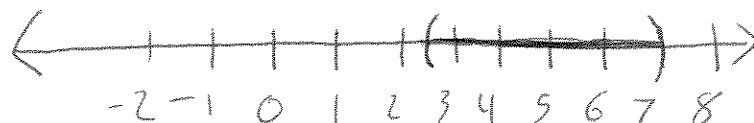
$$\begin{aligned} \Rightarrow 0 &\geq 10z \\ \Rightarrow 0 &\geq z \end{aligned}$$



$$\begin{array}{r} +2z \quad +2z \\ \hline \end{array}$$

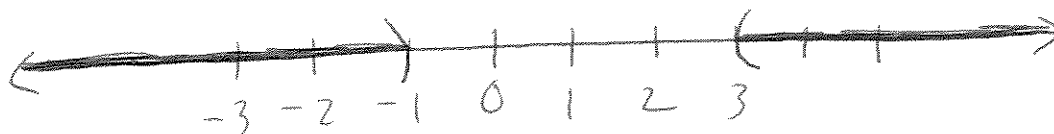
$$4. 0 < 2x - 5 < 9$$

$$\begin{aligned} 0 < 2x - 5 & \quad 2x - 5 < 9 \\ \Rightarrow 5 < 2x & \quad 2x < 14 \\ \Rightarrow \frac{5}{2} < x & \quad x < 7 \end{aligned}$$



The last example involves what is called a *compound inequality*. A compound inequality is built from two inequalities. If both inequalities must be satisfied, we combine them with the word "and". So, in the last example, we'd say $0 < 2x - 5$ and $2x - 5 < 9$.

On the other hand, suppose we wanted to describe the set of real numbers graphed below:



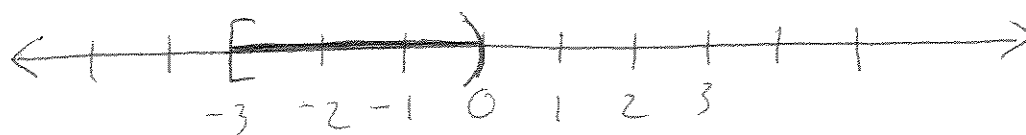
Here, we would say either $x < -1$ or $x > 3$. Note there's no way to express an "or" statement as a double inequality as we were able to do for an "and" statement.

We would also write down these ideas symbolically using set notation. The symbol $\cap$, which in mathematical language stands for *intersection*, can

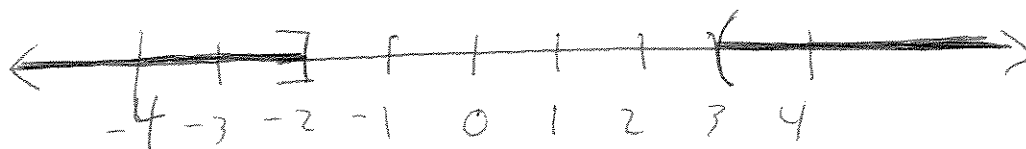
be used in place of the word "and", while the symbol $\cup$, which in mathematical language stands for *union*, can be used in place of the word "or".

Let's do some more examples.

Express the sets of numbers drawn below using set notation.



1. $\{x \mid x \geq -3\} \cap \{x \mid x < 0\}$



2. $\{x \mid x \leq -2\} \cup \{x \mid x > 3\}$