

Math 1010 - Lecture 6 Notes

Dylan Zwick

Fall 2009

1 Linear Equations and Problem Solving

In this lecture and the next, we'll be discussing how linear equations can be used to solve a number of common math problems, many with respect to real world applications. The goal here is for you to see many examples of linear equations, so you can get use to them, as well as learning some important ways in which these equations are used. Note that most of these equations will not be in our standard form $ax + b = 0$, but they can be converted to this form if necessary, and so are still linear equations.

1.1 Percent Problems

Rates that describe increases, decreases, and discounts are often given as percents. Percent means *per hundred*. Some examples are 10% means 10 per 100, and 34% means 34 per 100.

The primary use of percents is to compare two numbers. So, for example, we may say that 2 is twenty five percent (25%) of 8, or that 10 is two hundred percent (200%) of 5. In these examples we have a *base number*, and a number being compared with the base number. In the first example the base number is 8, and in the second the base number is 5. The basic percent equation is:

$$\text{Compared number} = \text{percent} \times \text{base number}.$$

Note that before doing any calculations we must convert the percent to either a decimal or a fraction. So, for example, if we had 25% our conversion would be:

$$25\% = 0.25 = \frac{1}{4}$$

and we'd have to use either 0.25 or $\frac{1}{4}$ in our percent equation. For example, in our expression 2 is 25% of 8, we'd write this as:

$$2 = 0.25 \times 8.$$

Examples:

1. What is 65% of 800?
2. 42 is 120% of what number?
3. 1650 is what percent of 5000?

1.2 Ratios and Unit Prices

A percent compares a number with 100. A *ratio* is a more generic rate that compares one number with another. If a and b are two quantities (numbers), then the ratio of a to b is a divided by b , or a/b . Note the order here. The ratio of a to b means a/b , and it's in general different than the ratio of b to a , which means b/a .

Examples:

1. Express 12 ounces to 20 ounces as a ratio.
2. Express 1 pint to 1 gallon as a ratio. (Note the units!)

A place where these type of comparisons show up in the real world is when we're dealing with a unit price. A unit price comes up when items are being purchased in bulk. You pay a price for a large quantity of items, and you want to know what that total cost translates to in unit price, which is how much you pay per individual item.

The formula for unit price is:

$$\text{Unit Price} = \text{Total price} / \text{Total units}.$$

Examples:

1. What is the unit price (in dollars per ounce) of a 64-ounce bottle of juice for \$1.89?

2. Determine which is the better buy:

- (a) A $10\frac{1}{2}$ -ounce package of cookies for \$1.79.
- (b) A 16-ounce package of cookies for \$2.39.

Note - Do this calculation the next time you order pizza. You'll find that almost without fail the best deal, in terms of cost per square-inch of pizza, is the largest pizza. This is especially true when you factor in that the ratio of crust to toppings decreases with the size of the pizza.

1.3 Solving Proportions

A *proportion* is a statement that equates two ratios. For example, if the ratio of a to b is the same as the ratio of c to d , then we'd write:

$$\frac{a}{b} = \frac{c}{d}.$$

An example of this would be the proportion:

$$\frac{2}{4} = \frac{3}{6}.$$

So, for example, if you heard "the proportion of people making above \$100,000 per year in Palo Alto is much higher than the proportion of people making above \$100,000 per year in Dallas" this would mean the number of people with high salaries in Palo Alto, divided by the population of Palo Alto, is higher than the number of people with high salaries in Dallas, divided by the population of Dallas. We note that there are still

almost certainly more total people in Dallas with high salaries, but this is just because Dallas is much larger than Palo Alto.

One place where these proportion problems frequently come up is in geometry, where you're frequently dealing with "similar", which means proportional, shapes.

Examples:

1. (A harder one!) - A man who is 6 feet tall walks directly toward the tip of the shadow of a tree. When the man is 75 feet from the tree, he starts forming his own shadow beyond the shadow of the tree. The length of the shadow of the tree beyond this point is 11 feet. Find the height h of the tree. (*Hint* - Draw a picture.)