Math 1010 - Lecture 5 Notes

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1 Linear Equations

The major focus of this chapter is linear equations: the first, most common, and simplest type of algebraic equation. This is the chapter where we start to move beyond basic arithmetic and into algebra.

1.1 Introduction to Equations

An equation is a statement that equates two algebraic expressions. So, for example, we could have $x = 12$, $4x - 7 = 5$, or $x^2 - 1 = x + 5$.

Solving an equation involving a variable means finding all values of the variable for which the equation is true. Such values are *solutions*, and we say these solutions *satisfy* the equation.

The *solution set* of an equation is the set of all solutions of the equation. If the solution set of an equation is all real numbers we call the equation an *identity*. In other words, it's always true. For example, $3x - 15 = 3(x - 5)$ is true for any value of x.

An equation that is not an identity is a *conditional equation*. For a conditional equation some numbers will satisfy the equation and others won't. Figuring out the numbers that satisfy when we evaluate the equation is one of the major problems in algebra.

Two equations that have the same set of solutions are *equivalent equations*. For example, $x = 5$ and $x - 5 = 0$. Now, we can take an equation and apply any of the following operations to obtain an equivalent equation:

1. Simplify Either Side:

   Example : $4x^2 - x^2 + 5 = 17 \Leftrightarrow 3x^2 + 5 = 17$.

2. Add the same thing to both sides:

   Example: $3x^2 + 5 = 17 \Leftrightarrow 3x^2 = 12$.

3. Multiply both sides by a number, or divide both sides by a non-zero number:

   Example: $3x^2 = 12 \Leftrightarrow x^2 = 4$.

4. Interchange sides:

   Example: $x^2 = 4 \Leftrightarrow 4 = x^2$.

1.2 Solving Linear Equations in Standard Form

A *linear equation* in standard form is an equation in the form:

$$ax + b = 0 \text{ where } a \neq 0.$$

This is also known as a *first-degree* equation.

Now, to solve these equations we need to *isolate* x by finding an equivalent equation in the form:

$$x = a$$

where a will be the number that satisfies our equation. Note that for any linear equation in standard form there will be one and only one number that satisfies it. We can find this number through the sequence of equivalent equations:

$$ax + b = 0 \quad \text{Subtract } b \text{ from both sides.}$$
$$ax = -b \quad \text{Divide both sides by } a.$$
$$x = -\frac{b}{a} \quad \text{Solution.}$$

Note that as $a \neq 0$ we knew there would be no problems in the second step.

Examples:

1. Solve the linear equation $-14x - 28 = 0$:

2. Solve the linear equation $10 - 6x = -5$:

1.3 Linear Equations in Nonstandard Form

Linear equations frequently occur in nonstandard form. That is, they appear as equations that are equivalent to a linear equation in standard form. For example, the equation:

$$2x + 5 = 3x - 2$$

is equivalent to the linear equation in standard form:

$$x - 7 = 0$$

and so would be a linear equation in nonstandard form. We can use our rules and methods for manipulating equations to solve these equations just as we did for linear equations in standard form.

Examples:

1. Solve the linear equation $3x - 1 = 2x + 14$:

2. Solve the linear equation $8(x-8)=24$:

When dealing with a linear equation that contains fractions, you should first clear the equation of fractions by multiplying each side by the least common denominator of the fractions.

Examples:

1. Solve the linear equation $\dfrac{8x}{5}-\dfrac{x}{4}=-3$:

2. Solve the linear equation $\dfrac{11x}{6}+\dfrac{1}{3}=2x$:

Finally, there are situations where an equation may appear to be linear, but when it's simplified it turns out not to be. That is, when it's simplified in turns out the coefficient a in the equation $ax+b$ is zero. In these situations our equation may have infinitely many solutions (it may be an identity) or it may have no solutions.

Examples:

1. Solve the equation $4y-3=4y$:

2. Solve the equation $4(2x-3)=8x-12$: