

Math 1010 - Lecture 4 Notes

Dylan Zwick

Fall 2009

1 Algebraic Expressions

We introduced algebraic expressions at the end of our last lecture. Today, we're going to talk about how to do some basic manipulations with these expressions, usually to make them simpler, and how to translate these types of expressions into words and vice-versa.

1.1 Combining Like Terms

One of the major ways we can simplify algebraic expressions is by combining like terms. Two terms in an algebraic expression are said to be *like terms* if they are both constant terms or if they both have the same variable factor. So, for example, in the algebraic expression:

$$x^2y^3 - 3x^2y + 5x + 7x^2y - 2x + 8y - 3 + 4xy + 8$$

the terms $-3x^2y$ and $7x^2y$ are like terms, as are the terms $5x$, $-2x$ and -3 , 8 . One way that we can simplify an algebraic expression is by combining like terms. So, for example, we can combine (add) the terms $-3x^2y$ and $7x^2y$ to get the term $4x^2y$. Doing this we'd simplify our above polynomial to:

$$x^2y^3 + 4x^2y + 5x - 2x + 8y - 3 + 4xy + 8.$$

Doing the same thing with our other like terms we could further simplify the polynomial to obtain:

$$x^2y^3 + 4x^2y + 4xy + 3x + 8y + 5.$$

Note that all of these polynomials are the same, in the same way that $12, 5 + 7$, and $1 + 2 + 3 + 6$ are the same.

Examples:

1. Simplify the expression $18z + 14z$:

$$32z$$

2. Simplify the expression $-2a + 4b - 7a - b$:

$$-9a + 3b$$

3. Simplify the expression $3a - 5ab + 9a^2 + 4ab - a$:

$$2a - ab + 9a^2$$

1.2 Removing Symbols of Grouping

Another way that we can sometimes simplify an expression is by removing symbols of grouping. We remove the innermost groupings first, combine like terms, and then continue moving out until there are no more groupings left.

A set of parentheses preceded by a minus sign can be removed if we change the sign of all the terms in the parentheses. For example:

$$x - (2x + 5) = x - 2x - 5 = -x - 5.$$

If there's a plus sign we can remove the parentheses without modification. For example:

$$x + (2x + 5) = x + 2x + 5 = 3x + 5.$$

We can also occasionally simplify an expression by applying the distributive law and then grouping like terms.

1. Use the Distributive Property to simplify the expression $x(5x + 2)$:

$$x(5x) + x(2) = 5x^2 + 2x$$

2. Use the Distributive Property to simplify the expression $y(-y + 10)$:

$$y(-y) + y(10) = -y^2 + 10y$$

3. Use the Distributive Property to simplify the expression $-6x(9x - 4)$:

$$-6x(9x) - 6x(-4) = -54x^2 + 24x$$

1.3 An Aside on Simplification

Now, it should be noted that simplifying an expression is in many ways more of an aesthetic, or at least practical, consideration, as composed to a formal mathematical one. It's based upon the form of an equation that is easiest for a human to understand and manipulate, and this is a fairly subjective criteria.

For example, there are those who would say

$$\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}}$$

is not simplified, but

$$\frac{3\sqrt{2} + 2\sqrt{3}}{6}$$

is. Why the second is more, and not less, simple than the first is beyond me. Similarly, it's more of a personal judgement which of the following equivalent algebraic expressions:

$$(x + 1)^2$$

and

$$x^2 + 2x + 1$$

is simpler. Now, this isn't to say that simplifying numbers and algebraic expressions is not useful. It's incredibly useful and necessary to understand. But it's useful in that it makes future computations much, much easier, and the human criteria of what forms are easiest to work with is the motivation driving the concept of simplification.

1.4 Evaluating Algebraic Expressions

When we evaluate an algebraic expression, we substitute numerical values for each of the variables in the expression. This gives us a number. You must make sure you do the same substitution for each of the occurrences of a variable.

For example, if we wanted to evaluate the algebraic expression

$$x^2 - 3x + 7$$

at $x = 2$ we'd plug in 2 for x to get

$$2^2 - 3(2) + 7 = 5.$$

Examples:

1. Evaluate $y^2 - y + 5$ at $y = 2$:

$$2^2 - 2 + 5 = 7$$

2. Evaluate $5 - \frac{3}{x}$ at $x = -6$:

$$5 - \frac{3}{-6} = 5 + \frac{1}{2} = \frac{11}{2}.$$

3. Evaluate $x^2 - xy + y^2$ at $x = 2, y = -1$:

$$2^2 - (2)(-1) + (-1)^2 = 4 + 2 + 1 = 7.$$

2 Constructing Algebraic Expressions

The idea behind constructing an algebraic expression is that we take a verbal expression or relation, and translate it into an algebraic expression. While this may seem a bit obvious, the idea that you can do this was in many ways the first big idea in algebra, and it took mankind a long time to figure it out!

2.1 Translating Phrases

The idea behind translating a phrase is to take a verbal description of a mathematical equation or relationship, and find the correct corresponding equation or relationship. We have to keep in mind when we do this that many words are synonyms for the same relation or operation. For example, saying the product of a and b is the same as saying a multiplied by b .

Examples:

1. Write “twelve more than a number n ” as an algebraic expression:

$$n + 12$$

2. Write “seven-fifths of a number n ” as an algebraic expression:

$$\frac{7}{5}n.$$

3. Write “the product of a number y and 10 is decreased by 35” as an algebraic expression:

$$10y - 35.$$

Now, we’re not always given a verbal description of the name of the variable. Sometimes, we have to come up with the variable name ourselves. For example, if we’re asked to write “the sum of 7 and a number” we’re not told what variable name we should use for the unknown number. In this situation, we just pick a variable name. So, we’d say our variable name is x , and write $7 + x$.

1. Write “the sum of 7 and twice a number, all divided by 8” as an algebraic expression:

$$\frac{7 + 2x}{8}.$$

2. Write “the absolute value of a quotient of a number and 4” as an algebraic expression:

$$\left| \frac{n}{4} \right|.$$

Now, as you might imagine, this can go the other way as well. That is, you can be given an algebraic expression and be asked to write it as a verbal phrase. As mentioned earlier, given the ambiguities of the English

language, there are frequently many ways of saying the same thing, and translating algebraic expressions is no exception.

So, for example, if we wanted to express $3n + 7$ verbally we could say “the product of 3 and a number n , increased by 7.” We could also say “the sum of 7 and 3 times a number n .” There are many other possibilities. As long as your verbal phrase is correct and unambiguous, it’s fine.

Examples:

1. Express $2 - 3x$ as a verbal phrase:

“Two subtract three times x .”

2. Express $8(x - 5)$ as a verbal phrase:

“Eight multiplied by the difference of x and five.”

3. Express $\frac{3 - n}{9}$ as a verbal phrase:

“Three subtract a number n , all divided by nine.”

2.2 Constructing a Mathematical Model

The ability to translate a verbal phrase into a mathematical model is one of the most useful and fundamental applications of algebra. The textbook lays out a three step process for creating a mathematical model:

1. Construct a verbal model that represents the problem situation.
2. Assign labels to all quantities in the verbal model.
3. Construct a mathematical model (in the cases we’ll deal with in the first chapter, these will all be algebraic expressions).

So, for example, if we wanted to represent the total amount of money (in cents) represented by m dimes and n quarters, our mathematical model would be:

$$10m + 25n.$$

Note here that, implicit in this equation, is an idea that comes up all the time in real world applications. Namely, you have to make sure your units work out correctly. In this example this means our final answer has to be in terms of cents. In other examples we have to make sure we don't do anything weird, like add seconds and meters.

Walking through what we mean for this example, we use the relation that there are 25 cents in a quarter, and 10 cents in a dime:

$$(m \text{ dimes}) \left(\frac{10 \text{ cents}}{\text{dime}} \right) + (n \text{ quarters}) \left(\frac{25 \text{ cents}}{\text{quarter}} \right)$$

The "dime" terms and the "quarter" terms cancel out, leaving us with two terms involving cents, which we can then add without concern.

Examples:

1. Write "the amount of wage tax due for a taxable income of I dollars that is taxed at a rate of 1.25%" as an algebraic expression:

$$I \times .0125 = .0125I.$$

2. Write "the sum of three consecutive integers, the first of which is n " as an algebraic expression. (Note - Consecutive integers are integers that follow each other as we count. So, for example, 17, 18, 19 are consecutive integers.):

$$n + (n + 1) + (n + 2) = 3n + 3.$$