

Math 1010 - Lecture 3 Notes

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1 Operations with Real Numbers

In our last lecture we covered some basic operations with real numbers like addition, subtraction and multiplication. This should all be material with which you're familiar, if not very familiar, but there are a few instances where you might need some review, especially when dealing with fractions and mixed numbers.

We'll begin today by finishing up operations with real numbers.

1.1 Multiplying Fractions

To multiply two fractions, you take the product of the numerators, and divide by the product of the denominators:

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}.$$

For example:

$$1. \left(-\frac{3}{8}\right) \left(\frac{11}{6}\right) = -\frac{3 \cdot 11}{8 \cdot 6} = -\frac{33}{48} = -\frac{11 \cdot 3}{16 \cdot 3} = -\frac{11}{16}.$$

$$2. \left(-\frac{5}{8}\right) \left(-\frac{4}{5}\right) = \frac{5 \cdot 4}{8 \cdot 5} = \frac{20}{40} = \frac{1}{2}.$$

1.2 Reciprocals

The reciprocal of a non-zero number is 1 divided by that number. Zero has no reciprocal. So, the reciprocal of a is $1/a$.

For example:

1. The reciprocal of 5 is: $\frac{1}{5}$.
2. The reciprocal of $\frac{3}{7}$ is: $\frac{7}{3}$.

1.3 Division of Real Numbers

We define division of real numbers in terms of multiplication and reciprocals. So, $a \div b$ is a multiplied by the reciprocal of b :

$$a \div b = a \cdot \frac{1}{b}.$$

Note that we're assuming you're already familiar with and comfortable with dividing whole numbers. Also note that division by zero is not allowed.

Examples:

1. $12 \div 4 = 3$
2. $\frac{3}{4} \div \frac{5}{7} = \left(\frac{3}{4}\right) \left(\frac{7}{5}\right) = \frac{21}{20}$

1.4 Positive Integer Exponents

Exponentiating a number by a positive integer n (also known as taking a number to the n th power) means multiplying that number by itself n times:

$$a^n = \underbrace{a \cdot a \cdots a \cdot a}_{n\text{-times}}$$

Examples:

$$1. 7 \cdot 7 \cdot 7 \cdot 7 = 7^4 = 2401$$

$$2. 4^3 = 4 \cdot 4 \cdot 4 = 64$$

1.5 Order of Operations

To evaluate an expression involving more than one operation, use the following order:

1. First do operations that occur within symbols of grouping.
2. Then evaluate powers.
3. Then do multiplications and divisions from left to right.
4. Finally, do additions and subtractions from left to right.

Really, the important things to remember here are that you do the stuff in parentheses before the stuff outside parentheses, and that you do multiplication and division before you do addition and subtraction. These can be particularly important when using a calculator.

For example, if you wanted to calculate:

$$\frac{3 + 4}{2 + 12}$$

and you plugged in:

$$3 + 4/2 + 12$$

your calculator would return the answer 17, which is not the right answer. The reason your calculator does this is that it first divides 4 by 2, and then adds 3, 2, and 12. The way to enter this into your calculator would be:

$$(3 + 4)/(2 + 12).$$

This way, your calculator would first do what's in the parentheses, namely the additions, and then do what's outside the parentheses, namely the division. Entering the equations into your calculator like this would return $1/2$ or $.5$, which would be correct.

Note that whenever there's a fraction like $\frac{1+3}{2+7}$ we treat it as if there's a parentheses (a grouping) of the elements in the numerator and the elements in the denominator.

Examples:

$$1. \ 5^2 + 2[9 - (18 - 8)] = 25 + 2[9 - 10] = 25 + 2[-1] = 25 - 2 = 23$$

$$2. \ \frac{3 + \frac{3}{4}}{\frac{1}{8}} = \frac{\frac{15}{4}}{\frac{1}{8}} = \left(\frac{15}{4}\right) \left(\frac{8}{1}\right) = 30$$

2 Properties of Real Numbers

The textbook lists a number of properties of the real numbers. I won't write them all down on the board, but here they are:

Commutative Property of Addition : $a + b = b + a$.

Commutative Property of Multiplication: $ab = ba$.

Associative Property of Addition: $(a + b) + c = a + (b + c)$.

Associative Property of Multiplication: $a(bc) = (ab)c$.

Distributive Property: $a(b + c) = ab + bc$; $(a + b)c = ac + bc$.

Additive Identity Property: $a + 0 = 0 + a = a$.

Multiplicative Identity Property: $a \cdot 1 = 1 \cdot a = a$.

Additive Inverse Property: $a + (-a) = 0$.

Multiplicative Inverse Property: $a \cdot \frac{1}{a} = 1, a \neq 0$.

Most of these properties are intuitively obvious when it comes to the real numbers and at this level it can seem like just a fancy way of stating the obvious, but these concepts become important in later mathematics when you start to deal with relations that don't have such nice properties. For example, three dimensional rotations. We won't spend too long focusing on them, but you should know them, and there are a few of these properties that frequently trip students up, so we'll go over them in some more detail.

2.1 The Distributive Property

There's a tendency in some students to not carry the distributive property all the way through. So, for example, a frequent mistake is something along these lines:

$$7(3 + 5) = 7(3) + 5 = 26. \text{ (NOTE - THIS IS WRONG!!!)}$$

Here, the multiplication is only carried through to the first element in the sum, and not the second. The correct answer is:

$$7(3 + 5) = 7(3) + 7(5) = 56.$$

This mistake is especially common when dealing with fractions.

Examples:

1. $3(5 - 2) = 3(5) - 3(2) = 15 - 6 = 9$

2. $4\left(\frac{5}{3} + \frac{2}{9}\right) = 4\left(\frac{5}{3}\right) + 4\left(\frac{2}{9}\right) = \frac{20}{3} + \frac{8}{9} = \frac{68}{9}$

$$3. \quad \frac{2}{3} \left(\frac{1}{8} + \frac{2}{3} \right) = \frac{2}{3} \left(\frac{1}{8} \right) + \frac{2}{3} \left(\frac{2}{3} \right) = \frac{1}{12} + \frac{4}{9} = \frac{3}{36} + \frac{16}{36} = \frac{19}{36}.$$

2.2 You Can't Divide By Zero!

Another things that trips people up, both in math 1010 and in later math classes, is that you absolutely cannot divide by zero. It doesn't make sense. So, just decide right now that it can't be done and that you're not going to do it.

An example of why it can't be done, and why it would lead to problems if it could be done, is illustrated here:

$$\begin{aligned} 3 \cdot 0 &= 2 \cdot 0 \text{ (true)} \\ \rightarrow \frac{3 \cdot 0}{0} &= \frac{2 \cdot 0}{0} \\ \rightarrow 3 &= 2. \text{ (false!)} \end{aligned}$$

So, there are good reasons why we can't divide by zero.

3 Algebraic Expressions

One of the major ideas, if not *the* major idea, in algebra is using a letter (frequently x) to denote an unknown. So, we use a symbol as a stand-in for a number. Sometimes we may not know that number and we want to solve for it. Other times we may be expressing a relationship and we want to give a formula that's valid for any input number. The letter used as a stand-in is called a *variable*. Real numbers are called *constants*, and if we combine variables and real numbers using addition, subtraction, multiplication, and division we form collections called *algebraic expressions*. Note that these are also frequently referred to as *rational functions*.

The *terms* of an algebraic expression are those parts that are separated by *addition*. So, for example, the terms in the algebraic expression:

$$3x^2 - 2x + 7$$

are $3x^2$, $-2x$, and 7. Note that we view $-2x$, and not $2x$ as a term because the above equation is the same as:

$$3x^2 + (-2x) + 7$$

and so we view it as adding $-2x$. The terms $3x^2$ and $-2x$ are called the *variable terms*, and the term 7 is called the *constant term*. The constant that appears in the term is called the *coefficient*. So, for example, the coefficient of the $3x^2$ term is 3.

Examples:

1. What are the terms in the algebraic expression $6x^3 - x + 2$.

$6x^3$, $-x$, and 2.

2. What is the coefficient of the only variable term in the algebraic expression $3x^4 - 21$?

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We'll be talking much more about algebraic expressions in our next class, and indeed in the rest of this course.