Math 1010 - Lecture 31 Notes

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In this, our final lecture, we'll go over some more properties of logarithms, and then we'll work through some problems involving logarithms.

1 Properties of Logarithms

We saw in our last lecture that the properties of exponentials:

(a) $a^0 = 1$

(b) $a^1 = a$

(c) $a^x = a^x$

corresponded to the properties of logarithms:

(a) $\log_a(1) = 0$

(b) $\log_a(a) = 1$

(c) $\log_a(a^x) = x$.

Well, similarly, the properties of exponentials:

(d) $a^m a^n = a^{m+n}$

(e) $\dfrac{a^m}{a^n} = a^{m-n}$

(f) $(a^m)^n = a^{mn}$

correspond to the properties of logarithms:

(d) $\log_a(uv) = \log_a(u) + \log_a(v)$

(e) $\log_a\left(\frac{u}{v}\right) = \log_a(u) - \log_a(v)$

(f) $\log_a(u^n) = n\log_a(u)$.

Therefore, the six big properties of logarithms are:

(a) $\log_a(1) = 0$

(b) $\log_a(a) = 1$

(c) $\log_a(a^x) = x$

(d) $\log_a(uv) = \log_a(u) + \log_a(v)$

(e) $\log_a\left(\frac{u}{v}\right) = \log_a(u) - \log_a(v)$

(f) $\log_a(u^n) = n\log_a(u)$.

Note that the first three can be derived from the last three, but they're so common and important that we single them out for notice.

Note that it is *not* in general possible to simplify $\log_a(u+v)$ and it is certainly *not* the case that this is equal to $\log_a(u) + \log_a(v)$. So, don't make this mistake!

2 Using the Properties

Examples

Use the properties of logarithms to evaluate the following.

1. $\log_5\left(\sqrt[3]{5}\right)$

2. $\log_3(54) - \log_3(2)$

3. $\log_4\left(\frac{3}{16}\right) + \log_4\left(\frac{1}{3}\right)$

4. $\ln\left(\frac{e^3}{e^2}\right)$

Use the properties of logarithms to *expand* the expressions.

1. $\log_3(11x)$

2. $\log_4[x^6(x+y)^2]$

3. $\ln(\sqrt{x(x+2)})$

Use the properties of logarithms to *contract* the expressions.

1. $\log_3(2) + \dfrac{1}{2}\log_3(y)$

2. $2[\ln(x) - \ln(x+1)]$

3. $\frac{1}{4}\log_6(x+1) - 5\log_6(x-4)$

3 Problems involving Logarithms

1. If you invest $\$1000$ in a savings account that generates 2% interest per year, compounded annually, how long will it takes for your investment to quintuple (increase to fives times its original amount)? Express your answer as a logarithm, you don't need to calculate it.

2. Solve the exponential equation $5000 = 2500e^{0.09t}$ for t to determine the number of years for an investment of $\$2500$ to double in value when compounded continuously at the rate of 9%.

3. Use Newton's Law of Cooling to solve the following forensics problem. Newton's law of cooling states that:

$$kt = \ln\left(\frac{T - S}{T_0 - S}\right)$$

where T is the temperature of a body (in degrees Fahrenheit), t is the number of hours elapsed, S is the temperature of the environment, and T_0 is the initial temperature of the body.

A corpse was discovered in a motel room at 10:00 P.M., and its temperature was $85°F$. Three hours later, the temperature of the corpse was $78°F$. The temperature of the motel room is a constant $65°F$.

(a) What is the constant k?

(b) Find the time of death using the fact that the temperature of the corpse at the time of death was $98.6°F$.

(c) What is the temperature of the corpse two hours after death?