Math 1010 - Lecture 22 Notes

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During the last couple of lectures, we've been learning about rational functions and how we can manipulate and simplify them. In this lecture, we'll discuss equations involving rational functions, and how we can solve them.

1 Solving Rational Equations

Suppose we've given the equation:

$$\frac{9}{25-y} = -\frac{1}{4}$$

and asked to find the value of y such that the equation is satisfied. How would we solve it?

Well, the first thing we do is we *MOVE EVERYTHING TO ONE SIDE!* I emphasize this because it's the first thing you'll want to do for all of these, so you need to get use to it. I'll repeat *move everything to one side*. We can do that in this situation by adding $1/4$ to both sides to get:

$$\frac{9}{25-y} + \frac{1}{4} = 0.$$

Now, we have a sum of rational functions, and we want to add them. So, in order to add them we need to find a common denominator. We can do this by multiplying the top and bottom of the first term by 4, and the top and bottom of the second term by $25 - y$. If we do this we get:

$$\frac{36}{4(25-y)} + \frac{25-y}{4(25-y)} = 0.$$

Now that we have a common denominator we can add the two terms to get:

$$\frac{61-y}{4(25-y)} = 0.$$

Now, we note that we'll have a division by 0 problem if $y = 25$, so we have to keep in mind that $y = 25$ is not a possible solution. Keeping this in mind if we then multiply both sides by $4(25-y)$ (remembering that anything multiplied by 0 is 0) we get:

$$61 - y = 0,$$

which we can solve to get out solution $y = 61$.

So, what were our steps here:

1. Move everything to one side.

2. Express everything on the side that isn't zero as a rational function.

3. Multiply both sides by the denominator of the rational function, keeping in mind that any solutions for which this denominator is zero is not allowed.

4. Solve the resulting algebraic equation.

Let's do some examples.

Examples

1. $\dfrac{2}{x-10} - \dfrac{3}{x-2} = \dfrac{6}{x^2 - 12x + 20}$.

$$\frac{2(x-2)}{(x-10)(x-2)} - \frac{3(x-10)}{(x-10)(x-2)} - \frac{6}{(x-10)(x-2)} = 0$$

$$\Rightarrow \frac{-x+20}{(x-10)(x-2)} = 0 \Rightarrow 20-x=0 \Rightarrow \boxed{x=20}$$

2. $\dfrac{x}{x-2} + \dfrac{3x}{x-4} = -\dfrac{2(x-6)}{x^2 - 6x + 8}$.

$$\frac{x(x-4)}{(x-2)(x-4)} + \frac{3x(x-2)}{(x-2)(x-4)} + \frac{2(x-6)}{(x-2)(x-4)} = 0 \Rightarrow \frac{4x^2 - 8x - 12}{(x-2)(x-4)} = 0$$

$$\Rightarrow 4x^2 - 8x - 12 = 0 \Rightarrow (4x-12)(x+1) = 4(x-3)(x+1) = 0$$

So, $\boxed{x=3 \text{ or } x=-1}$

3. $\dfrac{10}{x(x-2)} + \dfrac{4}{x} = \dfrac{5}{x-2}$.

$$\frac{10}{x(x-2)} + \frac{4(x-2)}{x(x-2)} - \frac{5x}{x(x-2)} = 0$$

$$\Rightarrow \frac{-x+2}{x(x-2)} = 0 \Rightarrow 2-x=0 \Rightarrow x=2$$

But wait! $x \neq 0$ and $x \neq 2$, so this isn't allowed, and there are $\boxed{\text{no solutions}}$

3

4. The cost C in dollars of removing $p\%$ of the air pollutants in the stack emissions of a utility company is modeled by the equation below. Determine the percent of air pollutants in the stack emissions that can be removed for $680,000$.

$$C = \frac{120,000p}{100-p}.$$

$680,000 = \frac{(120,000)p}{100-p}$

$\Rightarrow \frac{680,000(100-p)}{100-p} - \frac{120,000p}{100-p} = 0$

$\Rightarrow \frac{68,000,000 - 800,000p}{100-p} = 0$

$\Rightarrow p = \frac{68,000,000}{800,000}$

$= \frac{680}{8}$

$= \boxed{85}$

So, 85% of the pollutants.

2 Direct and Inverse Variation

If you're selling candy bars, and each candy bar sells for 2, then the relation between the total amount of revenue you make, R, and the number of candy bars you sell, N is:

$$R = 2N.$$

This is an example of something called *direct variation*. If you've got a relation of the form $y = kx$, or in words where the value of one quantity is a constant multiple of the value of another, then this relation is called direct variation, and we'd say that y varies directly with x.

You can also have a situation of the form $y = kx^n$ where n is some power, and we'd say that y is directly proportional to, or varies directly with, the nth power of x.

A relation of the form $y = k/x$ is an inverse relation, and we'd say that y varies inversely with x. For a relation of the form $y = k/x^n$ we'd say that y varies inversely with the nth power of x.

Finally, a relation of the form $z = kx^n y^m$ is called joint variation, and we'd say that z varies jointly with the nth power of x and the mth power of y.

Some examples of where these kinds of relations occur in the real world are:

- The voltage across a resistor is directly proportional to the current through the resistor. The constant of proportionality is called the *resistance* of the resistor, and the relation $V = IR$ is known as Ohm's law.

- If you drop an object from a building, the distance it falls (neglecting air resistance) is directly proportional to the *square* of the amount of time that it falls. The relation is of the form $d = (1/2)gt^2$, where g is the acceleration of gravity.

- If the temperature of a gas is not allowed to change, its absolute pressure P is inversely proportional to its volume V. This relation, $P = k/V$, is called *Boyle's law*.

Examples

1. Express as an equation the statement "C varies directly with r."

$$C = kr$$

2. Express as an equation the statement "A varies jointly with l and w."

$$A = klw$$

3. The total revenue R is directly proportional to the number of units sold x. When 500 units are sold, the revenue is $4,825. Find the revenue when 620 units are sold.

$$R = kx$$

$$\$4,825 = k(500) \implies k = \frac{4825}{500} = 9.65$$

So,

$$R = (9.65)(620) = \boxed{\$5,983}$$