Math 1010 - Lecture 21 Notes

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In this lecture we'll learn how to divide polynomials. The procedure for dividing polynomials turns out to be, more or less, exactly the same as the procedure for dividing integers.

1 Dividing Integers

The procedure for dividing integers, performed on a specific example, is given below:

$$
\begin{array}{r}
112 \\
9\,\overline{)\,1013} \\
-9 \\
\hline
11 \\
-9 \\
\hline
23 \\
-18 \\
\hline
5
\end{array}
\quad = 112 + 5/9
$$

So, we'd say the quotient is 112 and the remainder is 5.

2 Dividing Polynomials

Well, the same basic method works for dividing polynomials. For example, to divide $m^4 + 2m^2 - 7$ by m we'd get:

$$
\begin{array}{r l}
 & m^3 + 2m \\
m \,\big)\!\! & \overline{m^4 + 2m^2 - 7} \\
 & -m^4 \\
 & \overline{\quad\quad 2m^2} \\
 & \quad -2m^2 \\
 & \overline{\quad\quad\quad\quad -7}
\end{array}
\quad = \boxed{m^3 + 2m - \frac{7}{m}}
$$

Similarly, to divide $x^2 - 8x + 15$ by $x - 3$ we'd get:

$$
\begin{array}{r l}
 & x - 5 \\
x - 3 \,\big)\!\! & \overline{x^2 - 8x + 15} \\
 & -x^2 + 3x \\
 & \overline{\quad -5x + 15} \\
 & \quad +5x - 15 \\
 & \overline{\quad\quad\quad 0}
\end{array}
\quad = \cancel{(x-3)(x-5)} \quad \boxed{x - 5}
$$

Getting the hang of it?

Let's do some examples.

Examples

1. Divide $x^2 - 5x + 8$ by $x - 2$.

$$
\begin{array}{r l}
 & x - 3 \\
x - 2 \,\big)\!\! & \overline{x^2 - 5x + 8} \\
 & -(x^2 - 2x) \\
 & \quad -3x + 8 \\
 & -(-3x + 6) \\
 & \overline{\quad\quad\quad 2}
\end{array}
\quad = \boxed{(x - 3) + \frac{2}{x - 2}}
$$

2. Divide $5 + 4x - x^2$ by $1 + x$.

Convert to standard form.

$$\begin{array}{r l}
 & -x + 5 \\
x+1 \,\big) & -x^2 + 4x + 5 \\
 & -(-x^2 - x) \\
\hline
 & 5x + 5 \\
 & -(5x+5)
\end{array}$$

$= -x+5$ or $\boxed{5-x}$

3. $\dfrac{12x^2 + 17x - 5}{3x+2}$.

$$\begin{array}{r l}
 & 4x + 3 \\
3x+2 \,\big) & 12x^2 + 17x - 5 \\
 & -(12x^2 + 8x) \\
\hline
 & 9x - 5 \\
 & -(9x+6) \\
\hline
 & -11
\end{array}$$

$$\boxed{4x + 3 - \frac{11}{3x+2}}$$

4. $(8x^5 + 6x^4 - x^3 + 1) \div (2x^3 - x^2 - 3)$.

$$\begin{array}{r l}
 & 4x^2 + 5x + 2 \\
2x^3 - x^2 - 3 \,\big) & 8x^5 + 6x^4 - x^3 + 1 \\
 & -(8x^5 - 4x^4 - 12x^2) \\
\hline
 & 10x^4 - x^3 + 12x^2 + 1 \\
 & -(10x^4 - 5x^3 - 15x) \\
\hline
 & 4x^3 + 12x^2 + 15x + 1 \\
 & -(4x^3 - 2x^2 - 6) \\
\hline
 & 14x^2 + \cancel{2x} + 15x + 7
\end{array}$$

$$= \boxed{4x^2 + 5x + 2 + \frac{14x^2 + 15x + 7}{2x^3 - x^2 - 3}}$$

3 Synthetic Division

A trick that we can use to divide a polynomial, say a third order polynomial in the form $ax^3 + bx^2 + cx + d$ by $x - k$, is sketched below.

$$
\begin{array}{r|rrrr}
k & a & b & c & d \\
  &   & ka & \bigcirc & \bigcirc \\
\hline
  & a & b+ka & \bigcirc & r
\end{array}
\quad \leftarrow \text{Remainder}
$$

This is called synthetic division.

Examples

Calculate the following quotients using synthetic division.

1. $\dfrac{x^4 - 4x^3 + x + 10}{x - 2}$

$$
\begin{array}{r|rrrrr}
2 & 1 & -4 & 0 & 1 & 10 \\
  &   & 2 & -4 & -8 & -14 \\
\hline
  & 1 & -2 & -4 & -7 & -4
\end{array}
$$

$$\Rightarrow \boxed{x^3 - 2x^2 - 4x - 7 - \frac{4}{x-2}}$$

2. $\dfrac{x^3+3x^2-1}{x+4}$.

$$\begin{array}{r|rrrr} -4 & 1 & 3 & 0 & -1 \\ & & -4 & 4 & -16 \\ \hline & 1 & -1 & 4 & -17 \end{array}$$

$$\Rightarrow \boxed{x^2 - x + 4 - \frac{17}{x+4}}$$