Math 1010 - Lecture 21 Notes

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In this lecture we'll learn how to divide polynomials. The procedure for dividing polynomials turns out to be, more or less, exactly the same as the procedure for dividing integers.

1 Dividing Integers

The procedure for dividing integers, performed on a specific example, is given below:

$$\begin{array}{r} 112 \\ 9\overline{)1013} \\ -9 \\ \hline 11 \\ -9 \\ \hline 23 \\ -18 \\ \hline 5 \end{array} \qquad = 112 + 5/9$$

So, we'd say the quotient is 112 and the remainder is 5.

2 Dividing Polynomials

Well, the same basic method works for dividing polynomials. For example, to divide $m^4 + 2m^2 - 7$ by m we'd get:

$$
\begin{array}{r|l}
 & m^3 + 2m \\
\hline
m & m^4 + 2m^2 - 7 \\
 & -m^4 \\
\hline
 & \quad 2m^2 \\
 & \quad -2m^2 \\
\hline
 & \qquad -7
\end{array}
\qquad \boxed{m^3 + 2m - \frac{7}{m}}
$$

Similarly, to divide $x^2 - 8x + 15$ by $x - 3$ we'd get:

$$
\begin{array}{r|l}
 & x - 5 \\
\hline
x - 3 & x^2 - 8x + 15 \\
 & -(x^2 - 3x) \\
\hline
 & \quad -5x + 15 \\
 & \quad -(-5x + 15) \\
\hline
 & \qquad 0
\end{array}
\qquad = \boxed{x - 5}
$$

Getting the hang of it?

Let's do some examples.

Examples

1. Divide $x^2 - 5x + 8$ by $x - 2$.

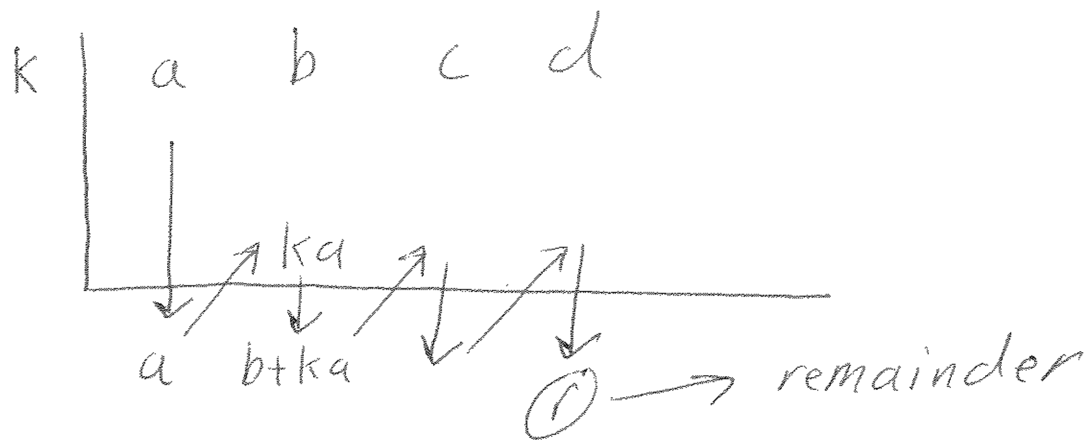
2. Divide $5 + 4x - x^2$ by $1 + x$.

3. $\dfrac{12x^2 + 17x - 5}{3x + 2}$.

4. $(8x^5 + 6x^4 - x^3 + 1) \div (2x^3 - x^2 - 3)$.

3 Synthetic Division

A trick that we can use to divide a polynomial, say a third order polynomial in the form $ax^3 + bx^2 + cx + d$ by $x - k$, is sketched below.



This is called synthetic division.

Examples

Calculate the following quotients using synthetic division.

1. $\dfrac{x^4 - 4x^3 + x + 10}{x - 2}$

2. $\dfrac{x^3 + 3x^2 - 1}{x + 4}$.