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# Math 1010 - Lecture 19 Notes

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In this lecture we'll define and discuss rational functions, and go over the basic operations of multiplication and division. We'll find that things work out pretty much the same way they do with our usual fractions we all know and love. The hardest thing is simplifying, as it will involve factoring.

## 1 Rational Functions

A rational function is a function made up of one polynomial divided by another. So, for example, the function  $f(x)$  below would be a rational function:

$$f(x) = \frac{x^2 + 2x + 5}{2x - 7}.$$

The domain of a rational function, which is the set of real numbers for which the function is defined, will be all real numbers for which the denominator is not zero. So, in our example the only number for which our denominator is zero is  $x = 7/2$ . So, the domain of the function  $f(x)$  is all real numbers not equal to  $7/2$ . We could write this as:

$$(-\infty, 7/2) \cup (7/2, \infty),$$

or just as  $x \neq 7/2$ .

### Examples

Find the domain of the following rational functions:

1.  $\frac{3x-3}{7}$ .

$7 \neq 0$ ,  
so all real numbers.

2.  $\frac{x^2+7}{x^2-4}$ .

$$x^2-4=0 \Rightarrow x=\pm 2.$$

So, all real numbers except  $\pm 2$ .

3.  $\frac{3x+11}{x^2+4}$ .

$$x^2+4 \geq 4$$

So,  $x^2+4 \neq 0$  for any  $x$ , and our domain is all real numbers.

Just as we can simplify fractions, if we have a common term in both our numerator and denominator we can simplify rational functions. The difference is that with rational functions these common terms are polynomials.

So, for example, if  $f(x)$ ,  $g(x)$ , and  $h(x)$  are polynomials with  $g(x) \neq 0$  and  $h(x) \neq 0$  then:

$$\frac{f(x)h(x)}{g(x)h(x)} = \frac{f(x)}{g(x)}.$$

In other words, we can cancel the function  $h(x)$  the same way we cancel a common multiple with fractions of integers. The only caveat here is the

points where  $h(x) = 0$ . Our old function will not be defined at these points, but our new function may be. So, to be correct, we require that our domain be the same as the domain of our old function.

For example:

$$\frac{x^2 + 4x + 4}{x^2 - 4} = \frac{(x+2)(x+2)}{(x+2)(x-2)} = \frac{x+2}{x-2}$$

with the caveat the our domain remains  $x \neq \pm 2$ .

*Examples*

Simplify the following rational functions.

1.  $\frac{2x-3}{4x-6}$

$$= \frac{\cancel{2x-3}}{2(\cancel{2x-3})} = \frac{1}{2} \quad x \neq \frac{3}{2}$$

2.  $\frac{x^2 - 7x}{x^2 - 14x + 49}$

$$\frac{x^2 - 7x}{x^2 - 14x + 49} = \frac{x(\cancel{x-7})}{(x-7)(\cancel{x-7})} = \frac{x}{x-7} \quad x \neq 7$$

$$3. \frac{x^4 - 25x^2}{x^2 + 2x - 15}$$

$$= \frac{x^2(x^2 - 25)}{(x+5)(x-3)} = \frac{x^2 \cancel{(x+5)}(x-5)}{\cancel{(x+5)}(x-3)}$$

$$= \frac{x^2(x-5)}{x-3} \quad x \neq 3, -5$$

## 2 Multiplying and Dividing Rational Functions

We multiply and divide rational functions in exactly the same way we multiply and divide fractions.

If  $f(x)$ ,  $g(x)$ ,  $h(x)$ ,  $w(x)$  are polynomials with  $g(x) \neq 0$  and  $w(x) \neq 0$  then:

$$\frac{f(x)}{g(x)} \cdot \frac{h(x)}{w(x)} = \frac{f(x)h(x)}{g(x)w(x)}$$

And similarly for division, where here we say  $h(x) \neq 0$  <sup>as well:</sup> ~~instead of  $w(x)$ :~~

$$\frac{f(x)}{g(x)} \div \frac{h(x)}{w(x)} = \frac{f(x)w(x)}{f(x)h(x)}$$

### Examples

Calculate the following:

$$1. \frac{(2x-3)(x+8)}{x^3} \cdot \frac{x}{3-2x}$$

$$= \frac{\cancel{(2x-3)}(x+8)x}{-x^3 \cancel{(2x-3)}} = - \frac{x(x+8)}{x^3 x^2}$$

$$= \boxed{-\frac{x+8}{x^2}} \quad \boxed{x \neq 0, \frac{3}{2}}$$

$$2. \frac{2t^2-t-15}{t+2} \cdot \frac{t^2-t-6}{t^2-6t+9}$$

$$= \frac{(2t+5)\cancel{(t-3)}}{\cancel{(t+2)}} \cdot \frac{\cancel{(t-3)}\cancel{(t+2)}}{\cancel{(t-3)}\cancel{(t-3)}} = \boxed{2t+5}$$

$$\boxed{t \neq -2, 3}$$

$$\cancel{t \neq -2, 3}$$

$$3. \frac{x^2+9}{5(x+2)} \div \frac{x+3}{5(x^2-4)}$$

$$= \frac{(x^2+9)}{\cancel{5(x+2)}} \cdot \frac{\cancel{5}\cancel{(x+2)}(x-2)}{x+3}$$

$$= \frac{(x^2+9)(x-2)}{(x+3)}$$

$$x \neq -2, -3, 2$$

$$4. \frac{(x^3y)^2}{(x+2y)^2} \div \frac{x^2y}{(x+2y)^3}.$$

$$= \frac{x^6y^2}{(x+2y)^2} \cdot \frac{(x+2y)^3}{x^2y}$$

$$= x^4y(x+2y)$$

$$x \neq 0,$$

$$y \neq 0,$$

$$x \neq -2y$$