

Math 1010 - Lecture 17 Notes

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Today we're going to delve into polynomials by first discussing the basic operations on polynomials: adding, subtracting, and multiplying. Division, which is more difficult, we'll be going over later. First, just to make sure we're all on the same page, here's the definition of a polynomial:

Definition. Let $a_n, a_{n-1}, \dots, a_1, a_0$ be real numbers and let n be a *non-negative integer*. A **polynomial in x** is an expression of the form:

$$a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$$

where $a_n \neq 0$. The polynomial is of **degree n** , and the number a_n is called the **leading coefficient**. The number a_0 is called the **constant term**.

So, for example, the following would be polynomials:

$$x^2 - 2x + 3, \quad x + 7, \quad 9,$$

while these would not:

$$\sqrt{x} - 1, \quad x^{-1} + 4, \quad x^2 + x - \sqrt{x} + 7.$$

1 Adding and Subtracting Polynomials

To add polynomials, we do it termwise. We can do this in two ways, which really turn out to be the same. We call these different ways horizontal and vertical.

To add polynomials horizontally, we just break them up in to their like terms, and add the like terms. So, for example, to add the polynomials:

$$(x^2 - 3x + 7) + (3x^2 + 12x - 7)$$

we combine the like terms, add them, and then combine the sums:

$$(x^2 + 3x^2) + (-3x + 12x) + (7 + (-7)) = 4x^2 + 9x + 0 = 4x^2 + 9x.$$

Subtraction works, more or less, the same way. You just add the opposite.

Examples

Calculate the following:

1. $(3x^3 - 2x + 7) + (4x^2 + 7x - 2)$

$$\begin{aligned} &= (3x^3) + (4x^2) + (-2x + 7x) + (7 + (-2)) \\ &= \boxed{3x^3 + 4x^2 + 5x + 5} \end{aligned}$$

2. $(2x^2 + 3x + 1) - (3x^2 + x - 7)$.

$$\begin{aligned} &= (2x^2 - 3x^2) + (3x + x) + (1 - (-7)) \\ &= \boxed{-x^2 + 4x + 8} \end{aligned}$$

The other way we can add polynomials is vertically. This is more or less the same way, but here you just line the like terms up in the same column, and add the coefficients in the column.

For example:

$$\begin{array}{r} 2x^3 - x^2 + 7x + 4 \\ x^3 + x^2 - 5x + 8 \\ \hline 3x^3 \qquad + 2x + 12 \end{array}$$

Examples

Add the following polynomials using the vertical method.

1. $3x^2 - 2$ and $4x^3 + 3x + 8$.

$$\begin{array}{r} 3x^2 - 2 \\ + 4x^3 + 3x + 8 \\ \hline \boxed{4x^3 + 3x^2 + 3x + 6} \end{array}$$

2. $2x^2 + 5x + 8$ and $x^2 - 2x - 4$.

$$\begin{array}{r} 2x^2 + 5x + 8 \\ + x^2 - 2x - 4 \\ \hline \boxed{3x^2 + 3x + 4} \end{array}$$

2 Multiplying Polynomials

To multiply two polynomials together you have to multiply *every* term in the first polynomial by *every* term in the second, and then add up (combine) all the terms you get to find your final answer. As with adding and subtracting polynomials, there is a horizontal and a vertical way.

When multiplying two polynomials together each with two terms we can use the acronym FOIL (First Inner Outer Last) to help us remember an order in which we can multiply. For example:

$$(x + 2)(x - 6) = x^2 + 2x - 6x - 12 = x^2 - 4x - 12.$$

Note that here we multiplied each term of the first by each term of the second, and then combined like terms. To do this vertically we'd write it as:

$$\begin{array}{r} x + 2 \\ x - 6 \\ \hline - 6x - 12 \\ x^2 + 2x \\ \hline x^2 - 4x - 12 \end{array}$$

Examples

Multiply the following polynomials using the horizontal method.

1. $x^2 + 3x - 2$ and $x^3 + 4x + 7$.

$$\begin{aligned} & (x^2 + 3x - 2)(x^3 + 4x + 7) \\ &= (x^2)(x^3) + (x^2)(4x) + (x^2)(7) + (3x)(x^3) + (3x)(4x) + (3x)(7) \\ & \quad + (-2)(x^3) + (-2)(4x) + (-2)(7) \\ &= x^5 + 4x^3 + 7x^2 + 3x^4 + 12x^2 + 21x - 2x^3 - 8x - 14 \\ &= x^5 + 3x^4 + 2x^3 + 14x^2 + 13x - 14 \end{aligned}$$

2. $x^2 - 3x + 1$ and $x + 5$.

$$\begin{aligned} & (x^2 - 3x + 1)(x + 5) = x^3 + 5x^2 - 3x^2 - 15x + x + 5 \\ &= x^3 + 2x^2 - 14x + 5 \end{aligned}$$

Multiply the following polynomials using the vertical method.

1. $x^2 - 2x + 1$ and $3x^2 + 4x + 5$.

$$\begin{array}{r}
 x^2 - 2x + 1 \\
 3x^2 + 4x + 5 \\
 \hline
 5x^2 - 10x + 5 \\
 4x^3 - 8x^2 + 4x \\
 3x^4 - 6x^3 + 3x^2 \\
 \hline
 \end{array}$$

2. $2x^2 - 5x + 6$ and $x - 1$.

$$\boxed{3x^4 - 2x^3 - 6x + 5}$$

$$\begin{array}{r}
 2x^2 - 5x + 6 \\
 x - 1 \\
 \hline
 -2x^2 + 5x - 6 \\
 2x^3 - 5x^2 + 6x \\
 \hline
 \end{array}$$

$$\boxed{2x^3 - 7x^2 + 11x - 6}$$