Math 1010 - Lecture 15 Notes

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In this lecture we'll go over another method for solving linear systems in two variables: the method of elimination. We'll then go over some applications of this method to systems of equations.

1 The Method of Elimination

Suppose we have a system of two equations with two unknowns:

$$\begin{array}{rcrcl} 3x & + & 2y & = & 4 \\ 5x & - & 2y & = & 8 \end{array}$$

Now, both sides of the equation are equal by definition, so we can add the left side of the first to the left side of the second, and the right side of the first to the right side of the second, and still get a legitimate equation. If we do this we get the equation:

$$8x = 12.$$

Solving this for x we get $x = 3/2$. If we then substitute this in for x in either of the two equations (either one will do, and either one will get you the same answer) we can solve for y. Taking equation 1 we get:

$$3\left(\frac{3}{2}\right) + 2y = 4$$
$$\rightarrow \frac{9}{2} + 2y = 4.$$

Solving this equation for y we get $y = -1/4$. So, the solution to our two equations is $x = 3/2, y = -1/4$ or the point $\left(\frac{3}{2}, -\frac{1}{4}\right)$.

Now, this first example problem was particularly easy, in that the first equation had a $2y$ term, and the second equation had a $-2y$ term, so adding them together let us eliminate the y terms without any manipulation. Unfortunately for us, for most equations we're not so lucky, and we have to do some manipulation before we can perform elimination. For example, if we're given the system of equations:

$$\begin{array}{rcrcr} 4x & - & 5y & = & 13 \\ 3x & - & y & = & 7 \end{array}$$

we cannot eliminate either the x or y variables by just adding the first equation to the second. However, if we multiply the bottom equation by -5 we get the equivalent equation:

$$-15x + 5y = -35$$

which we can add to the first and eliminate y. If we do this we get:

$$-11x = -22,$$

from which we get $x = 2$. Plugging this in for x and solving for y we get:

$$4(2) - 5y = 13,$$

for which if we solve for y we get $y = -1$. So, our solution to our system is $(2, -1)$.

In general, if we're given two equations and two unknowns, we want to pick a variable, find the least common multiple of the coefficients, and multiply both equations by the appropriate numbers so that the coefficients or our chosen variable are opposite. Then, we add the two equations together, eliminating our chosen variable. That's a fairly scary couple of sentences, but the idea is pretty straightforward. The trick is to just work through a few of them to get the hang of it, and then you'll find it's pretty easy.

Examples

1. $$\begin{array}{rcrcr} 5x & + & 2y & = & 7 \\ 3x & - & y & = & 13 \end{array}$$

$\Rightarrow 5x + 2y = 7$
$6x - 2y = 26$
$\Rightarrow 11x = 33 \Rightarrow x = 3$

$5(3) + 2y = 7$
$\Rightarrow 2y = -8$
$\Rightarrow y = -4$

$\boxed{(3, -4)}$

2. $$\begin{array}{rcrcr} 12x & - & 5y & = & 2 \\ -24x & + & 10y & = & 6 \end{array}$$

$24x - 10y = 4$
$-24x + 10y = 6$
$\Rightarrow 0 = 2$ X <u>Bad.</u>

$\boxed{\text{No solution}}$

3. $$\begin{array}{rcrcr} -2x & + & 3y & = & 9 \\ 6x & - & 9y & = & -27 \end{array}$$

$-6x + 9y = 27$

$6x - 9y = -27$

$\Rightarrow \quad 0 = 0$ $\quad \boxed{\text{Infinitely many solutions}}$

4. $$\begin{array}{rcrcr} x & - & y & = & -\frac{1}{2} \\ 4x & - & 48y & = & -35 \end{array}$$

$-4x + 4y = 2$ $\quad x = \frac{1}{4}$

$4x - 48y = -35$

$\Rightarrow -44y = -33$ $\quad \boxed{\left(\frac{1}{4}, \frac{3}{4}\right)}$

$y = \frac{3}{4}$

2 Applications

We'll start by solving, using systems of equations, the problem presented at the start of lecture 14.

Examples

1. A nursery wants to mix two types of lawn seed. Type 1 sells for \$12 per pound, and type 2 sells for \$20 per pound. To obtain 100 pounds of a mixture at \$14 per pound, how many pounds of each type of seed are needed?

$12x + 20y = 1400$ $\quad \Rightarrow -12x - 12y = -1200$

$x + y = 100$

$\Rightarrow 8y = 200 \Rightarrow y = 25$

$\boxed{x = 75, \quad y = 25}$

2. How many liters of a 20% alcohol solution must be mixed with a 60% solution to obtain 40 liters of a 35% solution?

$$.2x + .6y = (.35)40$$
$$x + y = 40$$

$$\Rightarrow .2x + .6y = 14$$
$$\Rightarrow -x - 3y = -70$$

$$\Rightarrow -2y = -30 \Rightarrow y = 15$$

$$x = 25$$

$$\boxed{\begin{array}{l} x = 25 \\ y = 15 \end{array}}$$