

Math 1010 - Lecture 14 Notes

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Fall 2009

In today's lecture we're going to talk about how we solve systems of two linear equations with two variables. We've seen these before, only under a different guise. For example, the mixture problem:

A nursery wants to mix two types of lawn seed. Type 1 sells for \$12 per pound, and type 2 sells for \$20 per pound. To obtain 100 pounds of a mixture at \$14 per pound, how many pounds of each type of seed are needed?

can be reformulated as a system of equations. If we say x is the amount of seed 1, and y is the amount of seed 2, then we get the system:

$$\begin{array}{rcrcrcrcrcl} x & + & y & = & 100 \\ 12x & + & 20y & = & 1400 \end{array} \cdot$$

Today we'll learn how to solve these types of systems.

1 Systems of Equations

The general form of a system of equations is:

$$\begin{array}{rcrcrcrcrcl} ax & + & by & = & c \\ dx & + & ey & = & f \end{array}$$

where a, b, c, d, e, f are numbers. A solution to this system is a pair of numbers (x_0, y_0) that satisfies both equations *simultaneously*. In other

words, a pair of numbers that satisfies each of the linear equations in the system. Usually for 2×2 systems like the ones we'll be dealing with there is only one solution, but there are two special cases, corresponding to lines with the same slope, where we could have either an infinite number of solutions or no solutions.

Examples

Check to see if the given pairs of numbers are solutions to the following system:

$$\begin{array}{rcl} 5x & - & 4y = 34 \\ x & - & 2y = 8 \end{array}$$

1. $(0, 3)$

2. $(6, -1)$

2 Solving Systems by Graphing

One way that we can solve systems of equations is by noticing that both equations in the system represent a line. So, if we graph the two lines, the solution to the system will be the point where the two lines cross. This will also let us see visually the two special situations mentioned earlier. Namely, if the two lines are distinct and parallel, then the system has no

solution. If the two lines are in fact the same line, then the system has an infinite number of solutions. We'll illustrate these with examples.

Examples

For each of the following systems, first graph the two lines and use the graphs to estimate where the two lines cross. Then, check to verify that this estimate is indeed a solution.

1.
$$\begin{array}{rcl} 2x & + & 4y = 8 \\ 4x & - & 3y = -6 \end{array}$$

2.
$$\begin{array}{rcl} -x & + & 5y = 15 \\ 2x & - & 10y = -7 \end{array}$$

$$3. \quad \begin{array}{rcl} 2x & + & 3y = 7 \\ 2x & - & 5y = -1 \end{array}$$

$$4. \quad \begin{array}{rcl} x & - & y = 9 \\ 2x & - & 2y = 18 \end{array}$$

3 Solving Systems of Equations by Substitution

A more precise way of solving systems of equations is by substitution. The basic method here is that we use one equation to solve for one variable, say y , in terms of the other variable x . Then we plug in this solution for y into the second equation to get an equation just in terms of x , and solve this equation for x . Once we've got this solution for x , we plug it into our equation for y in terms of x to solve for y . Writing this out in words might be a bit confusing, but the actual method is pretty straightforward, as will show up in our examples. We'll go over how to solve the problems

we solved graphically using substitution and see that we get the same answers, as we know we must.

Examples

Solve the following systems of equations using the substitution method.

1.
$$\begin{array}{rcl} 2x & + & 4y = 8 \\ 4x & - & 3y = -6 \end{array}$$

2.
$$\begin{array}{rcl} -x & + & 5y = 15 \\ 2x & - & 10y = -7 \end{array}$$

$$\begin{array}{rcl} 3. & 2x & + \quad 3y = 7 \\ & 2x & - \quad 5y = -1 \end{array}$$

$$\begin{array}{rcl} 4. & x & - \quad y = 9 \\ & 2x & - \quad 2y = 18 \end{array}$$