

Math 1010 - Lecture 12 Notes

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Suppose you have \$500 in a savings account, and you get a paycheck every month, and with each paycheck you add \$150 to your savings account. You're saving for a new violin, which costs \$2000. How many months will you have to save before you can afford the new violin?

This is an example of a linear equation, and we can express the relationship as:

$$S = 150t + 500$$

where t is measured in months. Now, this equation is the equation for a line in a particular form, namely slope-intercept form.

1 Slope-Intercept Form

The graph of the equation

$$y = mx + b$$

is a line, and this form of equation is called slope-intercept form. The number m is the slope of the line, and the number b is the y -intercept.

Examples

1. Convert the linear equation $2x - 3y = 6$ into slope-intercept form.

2. Graph the equation below, and state its slope and y -intercept.

$$y = 150x + 500$$

3. For what x -value in the above equation will we have $y = 2000$?

2 Parallel and Perpendicular Lines

Two distinct lines are parallel if they have the same slope. So, for example, the lines given by the below equations:

$$\begin{aligned}y &= 2x \\ \text{and} \\ y &= 2x + 3\end{aligned}$$

are parallel.

Two lines are perpendicular if they intersect at a right angle. The relation between their slopes is, if line 1 has slope m_1 and line 2 has slope m_2 , then if the two lines are perpendicular if they satisfy:

$$m_1 = -\frac{1}{m_2}.$$

For example, the lines $y = 2x + 3$ and $y = -\frac{1}{2}x + 1$ are perpendicular.

Examples

State whether the given lines are parallel, perpendicular, or neither.

1. $y = x + 7$ and $y = -x + 2$.

2. $y = 3x + 9$ and $y = 3x - 4$.

3. $y = 2x + 3$ and $y = -2x + 7$.

3 Equations of Lines

If we're given a point (x_1, y_1) and a slope m we can find a line with slope m that passes through the given point. The equation for the line will be:

$$y - y_1 = m(x - x_1).$$

Example

1. Find a line with slope 2 that passes through the point $(3, 4)$.

Slope-intercept form is a common way of writing the equation for a line. However, there is another form that we've seen before, called *general form*. An equation of the form:

$$ax + by + c = 0$$

describes a line, and is called a general form linear equation. This form is slightly more general than slope-intercept form, in that it can be used to represent vertical lines, but what it gains in generality it loses in clarity.

Examples

1. Graph the equation $2x + 3y + 7 = 0$ and rewrite it in slope-intercept form.

2. Graph the equation $2x + 4 = 0$.

3. Graph the equation $3y - 9 = 0$.

4 Graphs of Linear Equalities

So far we've just dealt with linear equations, or in other words linear equalities. The move from linear equalities to linear inequalities is pretty straightforward. For example, the set of points (x, y) that satisfy the inequality:

$$y > 2x + 3$$

will just be all the points above the line $y = 2x + 3$.

In general, the rules for this situation are:

- Convert the linear inequality into slope-intercept form. Draw the line as a dotted line.
- If the inequality is of the form $y < mx + b$ shade in the region below the line. If the inequality is of the form $y > mx + b$ shade in the region above the line.
- If the inequality is not strict, and so involves a relation $\leq$ or $\geq$, we use the same rules, only we also include the line itself in our permissible points. This means that instead of a dotted line, we draw a solid line.

Examples

1. Graph the inequality $y \leq 2x + 4$.

2. Graph the inequality $y > 3x + 1$.

3. Graph the inequality $y < x$.

The exception to this rule is when you're dealing with vertical or horizontal lines. A vertical line will be given as an equation in the form $x = b$, and a horizontal line will be given in the form $y = b$. In these situations the rules are:

- If it's a vertical line, draw the dotted line. If it's of the form $x > b$ shade in the region to the right of the line, if it's of the form $x < b$, shade in the region to the left. If it's not a strict inequality, and so is of the form $\leq$ or $\geq$ the same rules apply, only we use a solid line instead of a dotted line.
- For a horizontal line it's the same idea, except if it's of the form $y > b$ we shade in the region above the line, and if it's of the form $y < b$ we shade in the region below the line. Same rules apply for dotted line vs. solid line.

Examples

1. Graph the inequality $y \geq 3$.

2. Graph the inequality $x < -2$.