Math 1010 - Lecture 11 Notes

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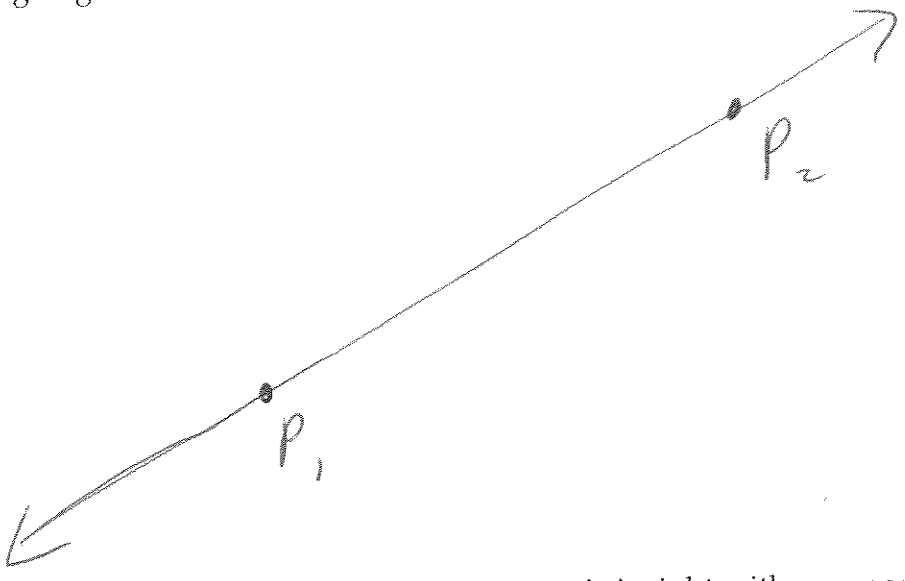
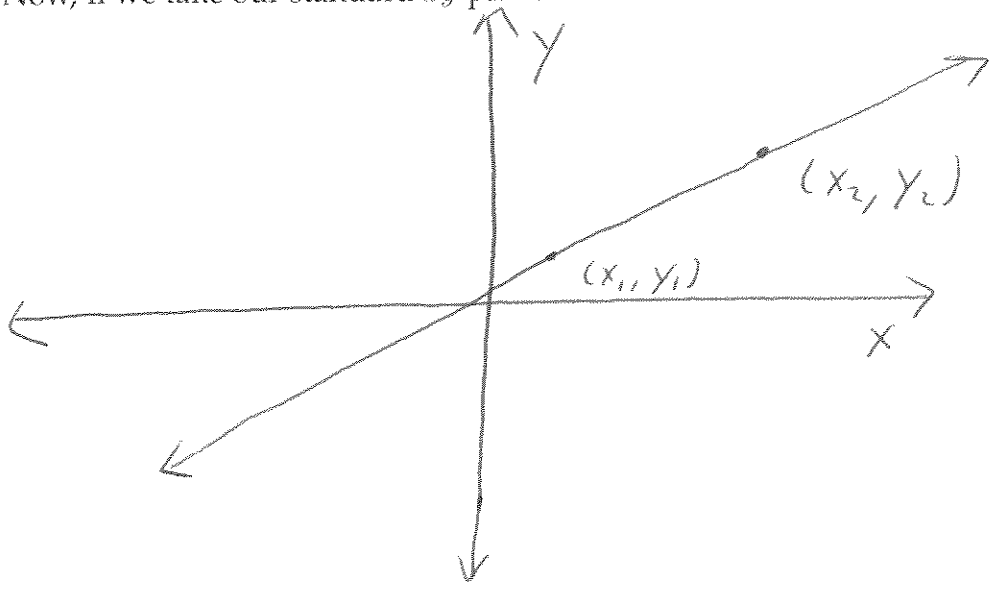
Today we're going to talk more about lines and how we graph them. Geometrically, if we have a plane, any two distinct points on the plane are going to describe a line:



We're going to combine this geometric insight with our more algebraic notions of a coordinate plane in order to talk about lines using algebraic equations.

The *slope* of a nonvertical line can be thought of as "rise over run". In other words, if you move to the right on the line one unit, how much do you move up or down? The slope tells you this. Sometimes when you're driving down a hill you might see a sign that says something like "6%". This means that for every hundred feet you move horizontally, you move down 6 feet vertically. This would be an example of a slope.

Now, if we take our standard xy-plane:



and two distinct points within that plane, then as we said before those two points are going to define a line. The slope of this line will be the change in y divided by the change in x. Put mathematically:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

where the variable m is used to represent slope. Note that for points (x_1, y_1) and (x_2, y_2) we could have also calculated the slope as:

$$m = \frac{y_1 - y_2}{x_1 - x_2}.$$

The point with which you start doesn't matter, as long as you're consistent. Meaning that if you've got $y_2 - y_1$ in your numerator, you better have $x_2 - x_1$ in your denominator, and if you've got $y_1 - y_2$ in your numerator, you'd better have $x_1 - x_2$ in your denominator. Switching them around, for example having $y_2 - y_1$ in your numerator and $x_1 - x_2$ in your denominator will get you the wrong answer (it will in fact give you the opposite of the right answer).

Examples

1. Find the slope of the line connecting the points $(2, 3)$ and $(-1, -2)$.

2. Find the slope of the line connecting the points $(-1, 2)$ and $(2, 1)$.

Now we note the following facts about the slope:

- A line with positive slope ($m > 0$) *rises* from left to right.
- A line with negative slope ($m < 0$) *falls* from left to right.
- A line with zero slope ($m = 0$) is *horizontal*.

- A line with undefined slope (when you try to calculate the slope, you divide by zero) is *vertical*.

Example

Take the example we went over at the end of the lectures notes for lecture 10, namely

$$2x - 3y = 6$$

and, given that its graph is a line, calculate the slope of the line and indicate if it's rising, falling, horizontal, or vertical.