

Math 1010 - Lecture 10 Notes

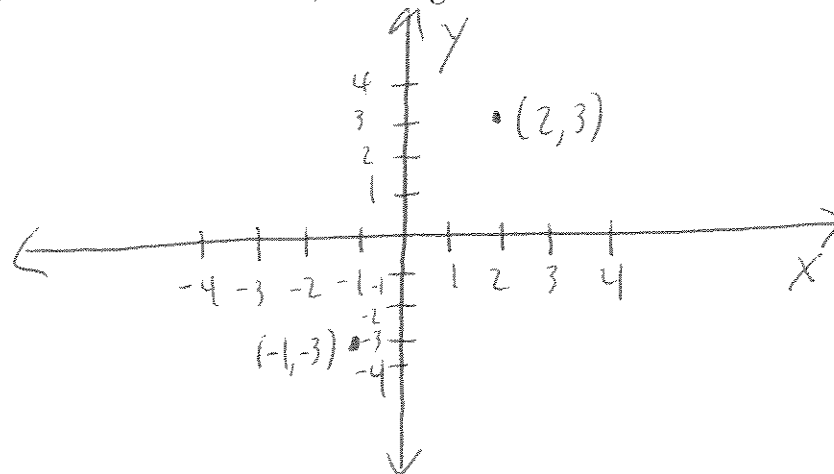
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Today we begin talking about graphs and functions. Graphs are a way of expressing relations between two variables pictorally, and functions are ways of expressing these relations mathematically.

1 The xy -coordinate system

We begin with a description of the xy -plane and the Cartesian coordinate system. You've all probably seen something like this before, either in a math class or in your day to day lives. The idea comes up all the time. Basically, you take two number lines, called *axes*, and you set them up perpendicular to each other, crossing at the zeros.



We typically call the horizontal axis the x -axis, and the vertical axis the y -axis. We can use these axes to describe points in the xy -plane. To specify a point, we give its x -coordinate and its y -coordinate. These coordinates,

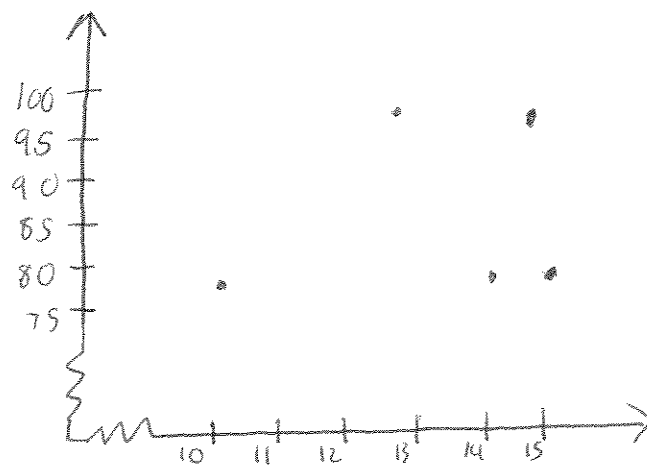
say for the point (a, b) , basically tell you “start from the origin and move a to the right, and then from there move b up”. So, for example, we’ve shown the point $(2, 3)$ in our plane above. It’s the number you get if you start from the origin and move two units to the right, and then three units up. Now, if these numbers are negative, say $(-1, -3)$ (also on our graph) this just says move -1 units to the right (which is the same as one unit to the left), and then -3 units up (which is the same as three units down). In this way we can specify points on our plane.

2 Representing Data

We can use this idea to visually represent data. For example, if we wanted to examine visually the correlation between quiz scores and exam scores, we could plot a graph with quiz scores on one axis, and exam scores on the other axis. Some points of data could be:

Quiz scores	13	14	15	15	10
Exam scores	96	78	93	78	78

and to graph these points we’d construct a graph that looks like this:



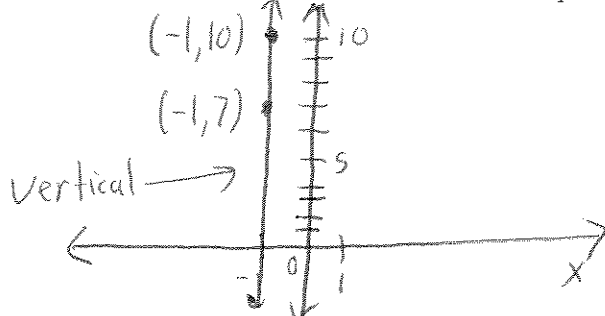
with quiz scores on the x -axis, and exam scores on the y -axis. We can do this for any type of relational data, and such a plot is called a *scatter plot*.

3 Distance Between Points and Midpoints

Suppose we have two points, for example $(-1, 7)$ and $(-1, 10)$ and we want to find the distance between them. Well, if the two lines are on the same vertical line, then the distance between the points will just be the absolute value of the difference of their y -values. If the two points are on the same horizontal line, then the distance between them will be the absolute value of the difference of their x -values.

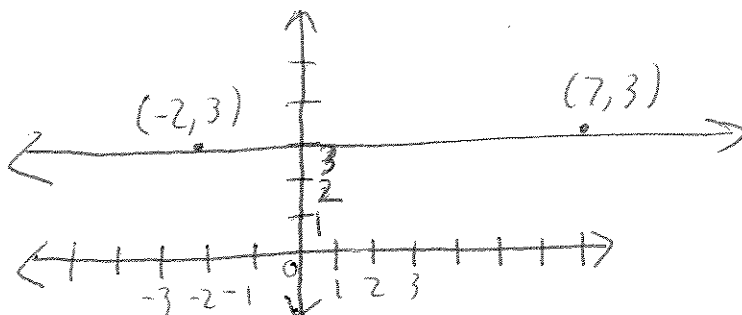
Examples

1. What is the distance between the points $(-1, 7)$ and $(-1, 10)$?



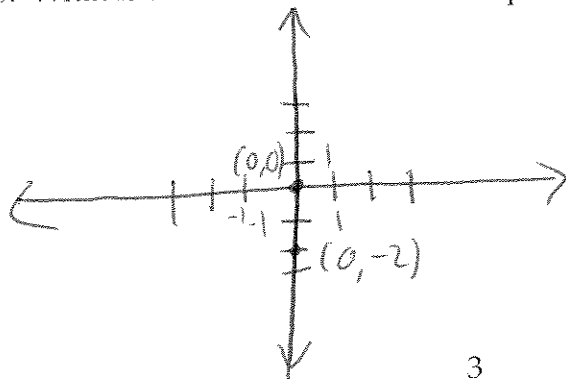
$$|10 - 7| = \boxed{3}$$

2. What is the distance between the points $(7, 3)$ and $(-2, 3)$?



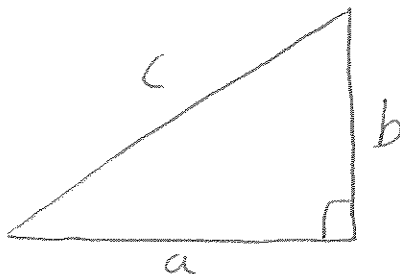
$$|-2 - 7| = \boxed{9}$$

3. What is the distance between the points $(0, 0)$ and $(0, -2)$?



$$|0 - (-2)| = |2| = \boxed{2}$$

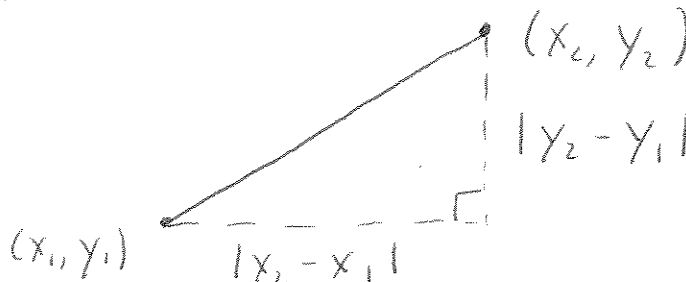
Now, suppose the two points are not on the same vertical or horizontal line. Then how do we find the distance between them? Well, we use something called the Pythagorean theorem. For a right triangle:



the Pythagorean theorem tells us that the length of the hypotenuse, c , is related to the lengths of the two sides, a and b , by the relation:

$$c^2 = a^2 + b^2.$$

Now, suppose we have two points (x_1, y_1) and (x_2, y_2) graphed like they are below:



The length of the horizontal side will just be the absolute value $|x_2 - x_1|$, and the length of the vertical side will be the absolute value $|y_2 - y_1|$. So, the Pythagorean theorem tells us the straight line distance will be:

$$c = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}.$$

Finally, if we note that $|x_2 - x_1|^2 = (x_2 - x_1)^2$, and same for the y terms, we get what's known as the *distance formula*:

$$c = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

Using this formula we can find the distance between any two points on the plane.

Examples

1. Find the distance between the points (3, 4) and (7, 7).

$$\begin{aligned} d &= \sqrt{(7-3)^2 + (7-4)^2} \\ &= \sqrt{4^2 + 3^2} = \sqrt{25} = \boxed{5} \end{aligned}$$

2. Find the distance between the points (1, 1) and (4, 5).

$$\begin{aligned} d &= \sqrt{(4-1)^2 + (5-1)^2} \\ &= \sqrt{3^2 + 4^2} = \boxed{5} \end{aligned}$$

Finally, between any two points we can find a midpoint. If we draw a straight line between our two points, the midpoint is on the line and equal distance from both points. The formula for finding the midpoint between two points (x_1, y_1) and (x_2, y_2) is:

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right).$$

Example

1. Find the midpoint between (1, 7) and (5, 3).

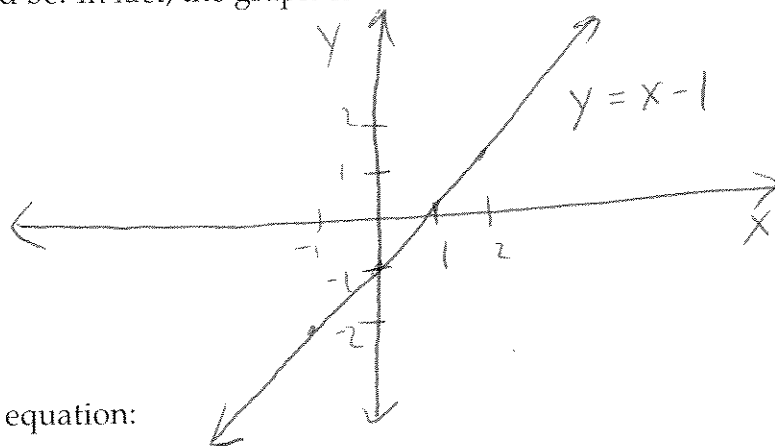
$$\left(\frac{1+5}{2}, \frac{7+3}{2} \right) = \boxed{(3, 5)}$$

4 Functions and Graphs

Up to this point we've only dealt with equations in one variable. Today, we're going to talk about equations with more than one variable, and how we can graph them. For any equation with two variables, x and y , the set of all values for x and y that solve the equation can be graphed. For example, suppose we have the equation:

$$y = x - 1.$$

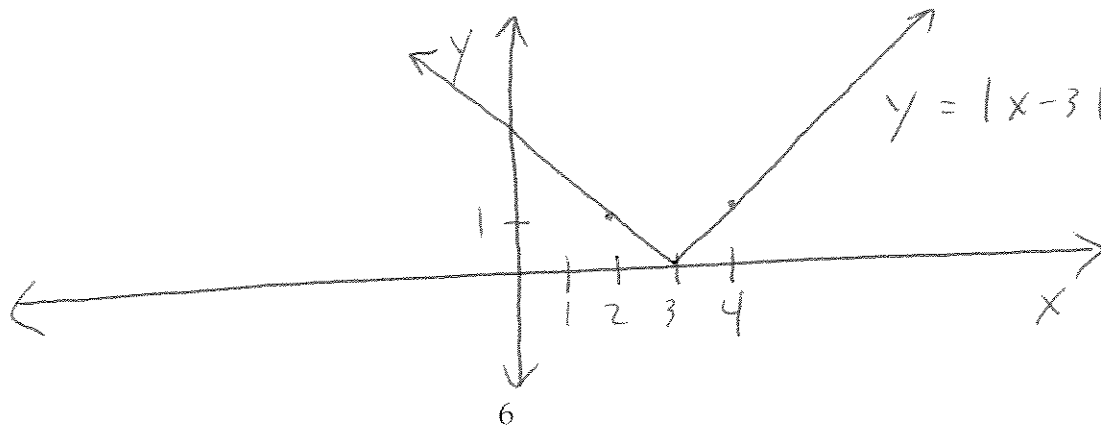
The point $(1, 1)$ would not be a solution to this, as $1 \neq 0$, but the point $(1, 2)$ would be. In fact, the graph of all the solutions would look like this:



For the equation:

$$y = |x - 3|$$

the graph would look like this:



Now, we'll be talking much more about graphing equations over the next few lectures, so this is just a quick overview. We should define some term:

- The y -intercept(s) of a graph is (are) where the graph touches the y -axis. It's when $x = 0$.
- The x -intercept(s) of a graph is (are) where the graph touches the x -axis. It's when $y = 0$. These are sometimes called "roots".

Example

1. What are the x and y intercepts of the graph of the equation:

$$2x - 3y = 6.$$

y -intercept $x = 0$

$$\Rightarrow -3y = 6 \Rightarrow \boxed{y = -2}$$

x -intercept $y = 0$

$$\Rightarrow 2x = 6 \Rightarrow \boxed{x = 3}$$

