

# Math 1010 - Practice Final Exam

University of Utah

Fall 2009

**Name:** Solutions

- There are 20 problems, and each is worth five points. So, there are 100 points possible.
- You are not allowed to get help from your textbook, class notes, other students, or any other form of outside aid. You're only allowed to use scratch paper if you need it. If you have questions, please ask your instructor. You may not talk with other students during the exam.

*Problem Scores:*

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20

**Total Score:**

1 **When Math had Numbers.** - Simplify the fraction:

$$\frac{3}{4} - \frac{\frac{1}{10}}{\frac{1}{5}}.$$

$$\begin{aligned}\frac{3}{4} - \frac{\frac{1}{10}}{\frac{1}{5}} &= \frac{3}{4} - \frac{\frac{1}{10}(5)}{\frac{1}{5}(5)} \\ &= \frac{3}{4} - \frac{5}{10} = \frac{3}{4} - \frac{1}{2} = \frac{1}{4}\end{aligned}$$

**2 Absolutely Equal.** Find the solution(s) to the equation:

$$|x + 8| = |2x + 1|.$$

There are two possibilities:

$$x + 8 = 2x + 1$$

$$\Rightarrow x = 7,$$

and

$$x + 8 = -2x - 1$$

$$\Rightarrow 3x = -9 \Rightarrow x = -3.$$

So, the solutions are:  $x = 7, -3$ .

- 3 **Working Together is Fun.** A master gravedigger can dig Ophelia's grave in five hours. An apprentice gravedigger can dig her grave in 8 hours. How long does it take the two of them to dig the grave if they work together, assuming they don't talk.

$$R_1 = \frac{1\text{grave}}{5\text{hours}}$$

$$R_2 = \frac{1\text{grave}}{8\text{hours}}$$

If  $R$  represents the work rate when they both work together we get  $R = R_1 + R_2$ .

Now, the time required to dig a grave is  $1/R$  where  $R$  is the work rate, and so we get:

$$t = \frac{1}{R} = \frac{1}{\frac{1}{5} + \frac{1}{8}} = \frac{40}{13} = 3\frac{1}{3}\text{hours}.$$

**4 Going the Distance.** Find the distance between the points  $(1, 3)$  and  $(4, -2)$ .

$$d = \sqrt{(4 - 1)^2 + (-2 - 3)^2} = \sqrt{3^2 + 5^2} = \sqrt{9 + 25} = \sqrt{34}$$

**5 Find that Line.** Find an equation for the line with slope 2 and  $x$ -intercept 5. Note that it says  $x$ -intercept, not  $y$ -intercept.

$$y = mx + b$$

The slope  $m$  is 2, so we get:

$$y = 2x + b.$$

We know that when  $x = 5$  we get  $y = 0$ , so plugging these values in we have:

$$0 = 2(5) + b \Rightarrow b = -10.$$

So, the equation for our line is  $y = 2x - 10$ .

**6 Master of Your Domain.** Find the domain of the function:

$$f(x) = \sqrt{x^2 - 1} + \frac{x - 2}{x^2 - x - 2}.$$

The domain of  $\sqrt{x^2 - 1}$  is  $x^2 - 1 \geq 0$ , and so either  $x \geq 1$  or  $x \leq -1$ .

The domain of  $\frac{x-2}{x^2-x-2}$  is all real numbers except those where the denominator is zero. The denominator factors as  $(x - 2)(x + 1)$ , and so the domain of the first term is all real numbers except  $x = 2$  and  $x = -1$ .

Combining these results, we get that our domain is  $x < -1$  or  $x \geq 1$  and  $x \neq 2$ .

The graph of this domain on the number line can be seen on the next page.

6 Master of Your Domain. Find the domain of the function:

$$f(x) = \sqrt{x^2 - 1} + \frac{x - 2}{x^2 - x - 2}$$

$$\sqrt{x^2 - 1} \quad \text{domain} \quad x \geq 1 \text{ or } x \leq -1$$

$$\frac{x - 2}{x^2 - x - 2} = \frac{x - 2}{(x - 2)(x + 1)} \quad \text{Domain } x \neq 2, x \neq -1$$

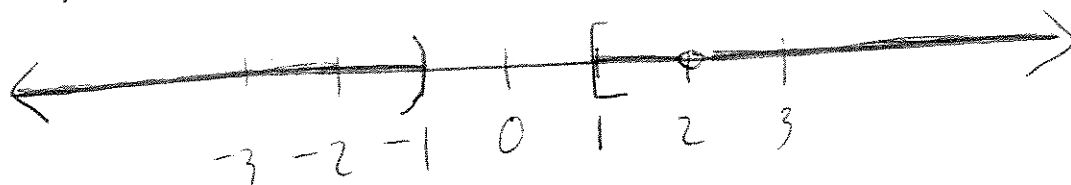
So, domain is:

$$x \geq 1 \text{ and } x \neq 2$$

or

$$x < -1$$

Graph:





**7 Working the System.** Solve the following system of linear equations:

$$\begin{array}{rcl} x & - & 2y = -1 \\ x & - & 5y = 2 \end{array}$$

The first equation gives us  $x = 2y - 1$ , and if we plug that in to the second equation we get  $2y - 1 - 5y = 2$ , and so  $-3y = 3$ , from which we get  $y = -1$ . If we then plug  $-1$  in for  $y$  we get  $x = 2(-1) - 1 = -3$ .

So,  $x = -3$  and  $y = -1$ .

8 **Not the O'reilly Factor.** Factor the following polynomial:

$$15x^2 - 11x + 2$$

$$(15x^2 - 11x + 2) = (5x - 2)(3x - 1).$$

9 **Keep It Simple.** Simplify the following expression. Your answer must contain only positive exponents:

$$\frac{(2x^{-2}y^4)^3y}{(10x^3y)^2}.$$

$$= \frac{2y^{11}}{25x^{12}}.$$

10 **Add 'Em Up.** Perform the indicated operations and simplify:

$$\begin{aligned} & \frac{x}{x^2 - 9} + \frac{3}{x^2 - 5x + 6} \\ &= \frac{x}{(x - 3)(x + 3)} + \frac{3}{(x - 3)(x - 2)} = \frac{x(x - 2)}{(x - 3)(x + 3)(x - 2)} + \frac{3(x + 3)}{(x - 3)(x + 3)(x - 2)} \\ &= \frac{x(x - 2) + 3(x + 3)}{(x - 3)(x + 3)(x - 2)} = \frac{x^2 + x + 9}{(x - 3)(x + 3)(x - 2)}. \end{aligned}$$

- 11 **Grow, Grow, Grow!** A biologist starts a culture with 100 bacteria. The population  $P$  of the culture is approximated by the model below, where  $t$  is the time in hours. Find the time required for the population to increase to 800 bacteria.

$$P = \frac{500(1 + 3t)}{5 + t}.$$

$$800 = \frac{500(1 + 3t)}{5 + t}$$

$$\Rightarrow 800(5 + t) = 500(1 + 3t)$$

$$\Rightarrow 4000 + 800t = 500 + 1500t$$

$$\Rightarrow 3500 = 700t$$

$$\Rightarrow 5 = t.$$

So,  $t = 5$  hours.

12 Thesis, Antithesis, Synthesis. Find the quotient and remainder:

$$\frac{x^5 - 13x^4 - 120x + 80}{x + 3}$$

Synthetic Division.

Cops!

~~$$\begin{array}{r|rrrrrr}
 -3 & 1 & -3 & 0 & 0 & -120 & 80 \\
 & & -3 & 18 & -54 & 162 & -126 \\
 \hline
 & 1 & -6 & 18 & -54 & 42 & -46
 \end{array}$$

$$x^4 - 6x^3 + 18x^2 - 54x + 42 -$$~~

$$\begin{array}{r|rrrrrr}
 -3 & 1 & -13 & 0 & 0 & -120 & 80 \\
 & & -3 & 48 & -144 & 432 & -936 \\
 \hline
 & 1 & -16 & 48 & -144 & 312 & -856
 \end{array}$$

$$13$$

$$x^4 - 16x^3 + 48x^2 - 144x + 312 - \frac{856}{x+3}$$

13 **Step One Is?**. Solve the rational equation:

$$\frac{15}{x} + \frac{9x - 7}{x + 2} = 9;$$

$$\Rightarrow \frac{15(x + 2)}{x(x + 2)} + \frac{x(9x - 7)}{x(x + 2)} - \frac{9x(x + 2)}{x(x + 2)} = 0;$$

$$\Rightarrow \frac{15x + 30 + 9x^2 - 7x - 9x^2 - 18x}{x(x + 2)} = 0;$$

$$\Rightarrow \frac{-10x + 30}{x(x + 2)} = 0;$$

$$\Rightarrow -10x + 30 = 0 \quad \text{Note: } x \neq -2, 0;$$

$$\Rightarrow 30 = 10x$$

$$\Rightarrow 3 = x.$$

$3 \neq 0$  and  $3 \neq -2$ , so this solutions works.

14 **Rules for Radicals** - Simplify the radical expression:

$$\begin{aligned} & -\sqrt[4]{\frac{42y^7}{81x^4}} \\ &= -\frac{\sqrt[4]{42y^7}}{\sqrt[4]{81x^4}} \\ &= -\frac{|y|\sqrt[4]{42y^3}}{3|x|} = -\frac{y\sqrt[4]{42y^3}}{3|x|}. \end{aligned}$$

Note we don't need to worry about the absolute value of  $y$  in the numerator as the  $\sqrt[4]{42y^3}$  term implies  $y \geq 0$ .



**15 Even More Radical** - Combine the radical expression, if possible:

$$3y\sqrt[4]{2x^5y^3} - x\sqrt[4]{162xy^7}.$$

$$= 3y|x|\sqrt[4]{2xy^3} - x|y|\sqrt[4]{(81)2xy^3}$$

$$= 3y|x|\sqrt[4]{2xy^3} - 3x|y|\sqrt[4]{2xy^3}.$$

Now for  $\sqrt[4]{2xy^3}$  to be well defined (for real numbers) we must have  $2xy^3 \geq 0$ . This will happen if either both  $x$  and  $y$  are negative, or both  $x$  and  $y$  are positive. In either case  $3y|x| = 3x|y|$ , and so the difference above is

$$3y|x|\sqrt[4]{2xy^3} - 3x|y|\sqrt[4]{2xy^3} = 0.$$

- 16 **It's Easy If You Try** - Write the complex number in standard form:  
(Note : Standard form is  $a + bi$ ).

$$\sqrt{-25} - \sqrt{49}$$

$$5i - 7 = -7 + 5i.$$

So,  $-7 + 5i$  would be the answer in standard form.

- 17 **The One Equation You Must Know!** - Find the solution(s) to the quadratic equation:

$$3x^2 + 7x = 2.$$

This equation is the same as  $3x^2 + 7x - 2 = 0$ . To find the solutions we use the quadratic formula:

$$x = \frac{-7 \pm \sqrt{7^2 - 4(3)(-2)}}{2(3)} = \frac{-7 \pm \sqrt{73}}{6}$$

- 18 **Real Graphs Have Curves** - Write the equation of the parabola with vertex  $(-2, 2)$  and  $y$ -intercept  $(0, -2)$ . Graph the parabola. (*Hint:  $x = -b/(2a)$* ).

Standard form is  $y = a(x - h)^2 + k$ , where the vertex has  $x$ -coordinate  $h$  and  $y$ -coordinate  $k$ .

So, the equation for our parabola is:

$$y = a(x - (-2))^2 + 2 = a(x + 2)^2 + 2.$$

Now, the  $y$ -intercept is when  $x = 0$ , so pluggint in  $x = 0$  we get:

$$y = a(2^2) + 2 = 4a + 2.$$

We're told the  $y$ -intercept is  $-2$ , and so

$$-2 = 4a + 2 \Rightarrow -4 = 4a \Rightarrow a = -1.$$

Therefore, the equation for our parabola is:

$$y = -(x + 2)^2 + 2.$$

Note that we could also write this as:

$$y = -x^2 - 4x - 2.$$

The graph is on the next page.

- 18 **Real Graphs Have Curves** - Write the equation of the parabola with vertex  $(-2, 2)$  and  $y$ -intercept  $(0, -2)$ . Graph the parabola. (Hint:  $x = -b/(2a)$ ).

Standard form:

$$\cancel{y = a(x-b)^2 + c} \quad y = a(x-h)^2 + k$$

Vertex is  $x = h$ , so  $h = -2$

$$y = a(x+2)^2 + k$$

Vertex is  $y = k$ , so  $k = 2$

$$\Rightarrow y = a(x+2)^2 + 2$$

At  $x = 0$ ,

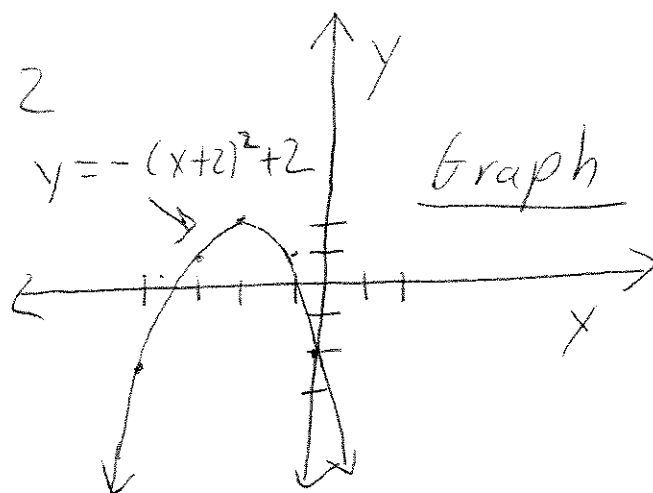
$$-2 = a(2)^2 + 2$$

$$\Rightarrow -4 = 4a$$

$$\Rightarrow a = -1.$$

so,

$$\boxed{y = -(x+2)^2 + 2}$$



19 **It's Logarithm, Logarithm, It's Big It's Heavy It's A Function**<sup>1</sup> -  
Compute the following logarithms:

- $\log_4(256)$

$$256 = 4^4, \text{ and so } \log_4(256) = \log_4(4^4) = 4 \log_4(4) = 4.$$

- $\log_2(0.25)$

$$\log_2(0.25) = \log_2(1/4) = \log_2(1) - \log_2(4) = 0 - \log_2(2^2) = -2 \log_2(2) = -2.$$

- $\log_{16}(8)$  (*Hint: Use the change of base formula.*)

$$\log_{16}(8) = \frac{\log_2(8)}{\log_2(16)} = \frac{\log_2(2^3)}{\log_2(2^4)} = \frac{3}{4}.$$

Narf! Zoit! Egad!

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<sup>1</sup>This is a very obscure and butchered reference to Ren and Stimpy.

- 20 **Ben Franklin Would Be Proud** - You invest money at an annual interest rate of 7 percent. How long (in years) will it take for your investment to triple (increase to three times its original amount)? Give your answer as a logarithmic expression, don't worry about a decimal value.

$$3P = P(1.07)^t$$

$$\Rightarrow 3 = (1.07)^t$$

$$\Rightarrow \log_{10}(3) = \log_{10}(1.07^t) = t \log_{10}(1.07)$$

$$\Rightarrow t = \frac{\log_{10}(3)}{\log_{10}(1.07)}.$$

This would be the correct answer for the exam. Note that the base of 10 in  $\log_{10}$  here was arbitrary, and we could have gone with any  $\log_a$  where  $a$  is a positive number that is not 1 and we would have been correct.

If you're curious,  $\frac{\log_{10}(3)}{\log_{10}(1.07)} = 16.238$ . So, 16.238 years.