

# Math 1010 - Final Exam

University of Utah

Fall 2009

Name: Solutions

- There are 20 problems, and each is worth five points. So, there are 100 points possible.
- You are not allowed to get help from your textbook, class notes, other students, or any other form of outside aid. If you have questions, please ask your instructor. You may not talk with other students during the exam.

*Problem Scores:*

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20

Total Score: 100

1 Simplifying Fractions. - Simplify the fraction:

$$\frac{\frac{2}{5} + \frac{1}{2}}{\frac{3}{5} - \frac{1}{3}}$$

$$\frac{\frac{2}{5} + \frac{1}{2}}{\frac{3}{5} - \frac{1}{3}} = \frac{\frac{4}{10} + \frac{5}{10}}{\frac{9}{15} - \frac{5}{15}} = \frac{\frac{9}{10}}{\frac{4}{15}}$$

$$\Rightarrow \frac{\frac{9}{10} (15)}{\frac{4}{15} (15)} = \frac{\frac{135}{10} (10)}{4 (10)} = \frac{135}{40}$$

$$= \frac{5(27)}{5(8)} = \boxed{\frac{27}{8}}$$

2 Solving Linear Equations with Absolute Values. Solve the equation:

$$|7 - 2x| = 15.$$

Two solutions

$$7 - 2x = 15$$

$$\Rightarrow -2x = 8$$

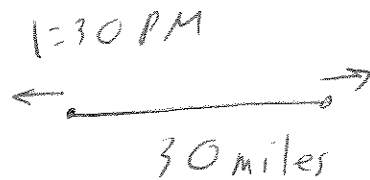
$$\Rightarrow \boxed{x = -4}$$

$$7 - 2x = -15$$

$$\Rightarrow -2x = -22$$

$$\Rightarrow \boxed{x = 11}$$

- 3 **Mass Transit.** A train leaves its station at 1:00 PM traveling with a constant speed of 60 miles per hour. Another train leaves the same station at 1:30 PM headed in the opposite direction traveling with a constant speed of 70 miles per hour. Solve an equation to decide at what time they will be 290 miles apart.



$$130t + 30 = 290$$

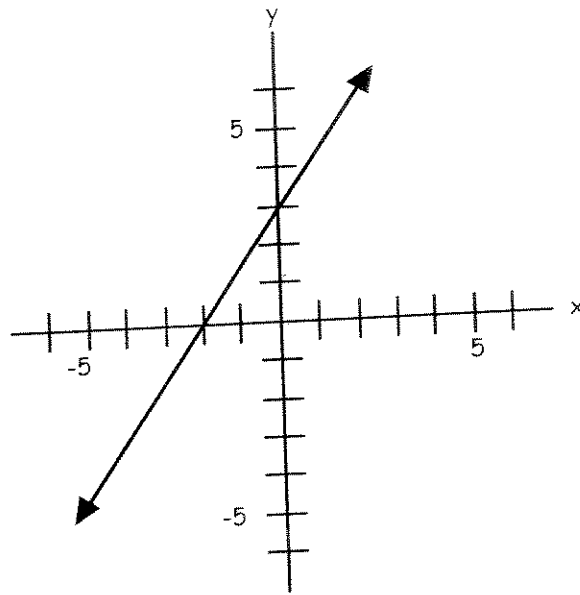
$$\Rightarrow 130t = 260$$

$$\Rightarrow t = 2 \text{ hours}$$

$$1:30 + 2 \text{ hours}$$

$$= \boxed{3:30 \text{ PM}}$$

4. Write an equation of the line shown:



$$\text{slope} = \frac{3}{2}$$

$$y\text{-intercept} = 3$$

$$y = \frac{3}{2}x + 3$$

- 5 **Find that Line.** Find an equation for the line passing through the point  $(6, -1)$  that is perpendicular to the line  $3x + 2y = 7$ .

$$3x + 2y = 7$$

$$\Rightarrow y = -\frac{3}{2}x + \frac{7}{2}$$

perpendicular slope is  $-\left(\frac{1}{-\frac{3}{2}}\right)$   
 $= \frac{2}{3}$

$$-1 = 6\left(\frac{2}{3}\right) + b$$

$$\Rightarrow -1 = 4 + b \quad \Rightarrow b = -5$$

$$\Rightarrow \boxed{y = \frac{2}{3}x - 5}$$

6 **Domains.** Find the domain of the function:

$$f(x) = \sqrt{10 - 2x}.$$

If  $x \leq 5$  then  $10 - 2x \geq 0$ .

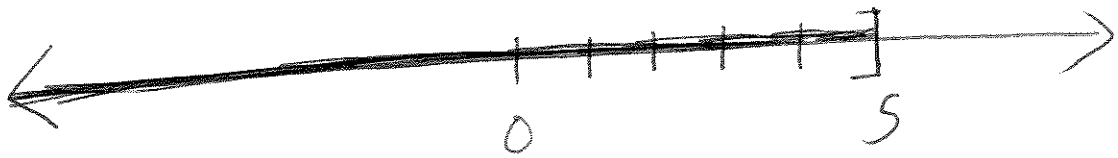
$$10 - 2x \geq 0$$

$$\Rightarrow 10 \geq 2x$$

$$\Rightarrow 5 \geq x$$

So,

$$x \leq 5$$



7 **Systems of Equations.** Solve the following system of linear equations:

$$\begin{aligned} 4x + 3y &= 8 \\ x - 2y &= 13 \end{aligned}$$

$$4x + 3y = 8$$

$$x - 2y = 13$$

$$\Rightarrow 4x + 3y = 8$$

$$-4x + 8y = -52$$

$$\Rightarrow 11y = -44 \Rightarrow y = -4$$

$$x - 2(-4) = 13$$

$$\Rightarrow x + 8 = 13$$

$$\Rightarrow x = 5$$

$$\boxed{\begin{aligned} x &= 5 \\ y &= -4 \end{aligned}}$$



8 **Solving Polynomial Equations.** Solve the following equation (that is to say, find the values of  $x$  that make the equality true):

$$x^2 - 15 = -2x$$

$$x^2 - 15 = -2x$$

$$\Rightarrow x^2 + 2x - 15 = 0$$

$$\Rightarrow (x+5)(x-3) = 0$$

$$\Rightarrow \boxed{x = 3 \text{ or } x = -5}$$

9 **Simplifying Polynomials.** Perform the indicated operations and simplify:

$$(a) (3x^5 + 4x^2 - 8x + 12) - (2x^5 + 4x^2 - 9)$$

$$= 3x^5 - 2x^5 + 4x^2 - 4x^2 - 8x + 12 - (-9)$$

$$= \boxed{x^5 - 8x + 21}$$

$$(b) (5x^3 + 4x - 3)(4x - 5)$$

$$(5x^3 + 4x - 3)(4x - 5)$$

$$20x^4 - 25x^3 + 16x^2 - 20x - 12x + 15$$

$$= \boxed{20x^4 - 25x^3 + 16x^2 - 32x + 15}$$

- 10 **Simplifying Rational Expressions.** Multiply and simplify the following rational expression:

$$\frac{x^2 - 16}{x^2 + 8x + 16} \cdot \frac{x^2 + x - 6}{x^2 - 6x + 8}$$

$$= \frac{\cancel{(x+4)}\cancel{(x-4)}}{\cancel{(x+4)}(x+4)} \cdot \frac{(x+3)\cancel{(x-2)}}{\cancel{(x-4)}\cancel{(x-2)}}$$

$$= \boxed{\frac{x+3}{x+4}} \quad \boxed{x \neq -4, 2, 4}$$

↑  
Domain

11 **Long Division.** Find the quotient and remainder using long division:

$$\frac{x^3 + 4x^2 + 7x + 7}{x^2 + 2x + 3}$$

$$\begin{array}{r} x + 2 \\ x^2 + 2x + 3 \overline{) x^3 + 4x^2 + 7x + 7} \\ \underline{- x^3 - 2x^2 - 3x} \phantom{+ 7} \\ 0 \phantom{x^3} 2x^2 + 4x + 7 \\ \underline{- 2x^2 - 4x - 6} \\ 0 \phantom{x^3} 0 \phantom{x^2} 8x + 13 \end{array}$$

$$= \boxed{x + 2 + \frac{1}{x^2 + 2x + 3}}$$

12 Solving Rational Equations. Solve the rational equation:

$$\frac{8}{3x+5} = \frac{1}{x+2}$$

$$\Rightarrow \frac{8}{3x+5} - \frac{1}{x+2} = 0$$

$$\frac{8(x+2)}{(3x+5)(x+2)} - \frac{(3x+5)}{(3x+5)(x+2)} = 0$$

$$\Rightarrow \frac{8x+16-3x-5}{(3x+5)(x+2)} = 0$$

$$\Rightarrow \frac{5x+11}{(3x+5)(x+2)} = 0$$

$$x \neq -\frac{5}{3}, -2$$

$$\Rightarrow 5x+11=0$$

$$\Rightarrow \boxed{x = -\frac{11}{5}}$$

- 13 **Simplifying Radical Expressions** - Simplify (i.e. write over a common denominator without a radical in the denominator) the expression:

$$\frac{x}{\sqrt{3x}} + \sqrt{27x}$$

and state the domain.

$$= \frac{x}{\sqrt{3x}} + \frac{\sqrt{27x} \sqrt{3x}}{\sqrt{3x}}$$

$$= \frac{x + \sqrt{81x^2}}{\sqrt{3x}} = \frac{x + 9x}{\sqrt{3x}}$$

$$= \frac{10x}{\sqrt{3x}} = \frac{10 \cancel{x} \sqrt{3x}}{3 \cancel{x}}$$

$$= \boxed{\frac{10}{3} \sqrt{3x}}$$

$$\text{Domain} = \boxed{x > 0}$$

Note: It's not  $x \geq 0$ .  $x$  cannot be 0.

14 Solving Radical Equations - Find the solution(s) to the equation:

$$\sqrt{3x+7} = x+3.$$

$$(\sqrt{3x+7})^2 = (x+3)^2$$

$$\Rightarrow 3x+7 = x^2+6x+9$$

$$\Rightarrow 0 = x^2+3x+2$$

$$\Rightarrow 0 = (x+2)(x+1)$$

$x = -2, -1$  are possible solutions.

(check:

$$x = -2$$

$$\sqrt{3(-2)+7} = \sqrt{1} = 1$$

$$(-2)+3 = 1 \quad \checkmark$$

$$x = -1$$

$$\sqrt{3(-1)+7} = \sqrt{4} = 2 \quad \checkmark$$

$$(-1)+3 = 2$$

So,  $\boxed{x = -1 \text{ or } x = -2}$

15 **Complex Numbers** - Write the quotient in standard form (i.e.  $a + bi$ ):

$$\frac{20}{3+i}$$

$$\left( \frac{20}{3+i} \right) \left( \frac{3-i}{3-i} \right)$$

$$= \frac{60-20i}{9+1} = \frac{60-20i}{10}$$

$$= \boxed{6-2i}$$



16 Quadratic Equations - Find the solution(s) to the quadratic equation:

$$6x^2 + 3x = 9.$$

$$6x^2 + 3x - 9 = 0$$

$$\frac{-3 \pm \sqrt{3^2 - 4(6)(-9)}}{2(6)}$$

$$= \frac{-3 \pm \sqrt{9 + 216}}{12}$$

$$= \frac{-3 \pm \sqrt{225}}{12} = \frac{-3 \pm \sqrt{5}}{12}$$

$$= \frac{-18}{12} \text{ or } \frac{12}{12}$$

$$\Rightarrow \boxed{-\frac{3}{2} \text{ or } 1}$$

17 **Graphing a Parabola** - Find the vertex of the parabola given by the equation:

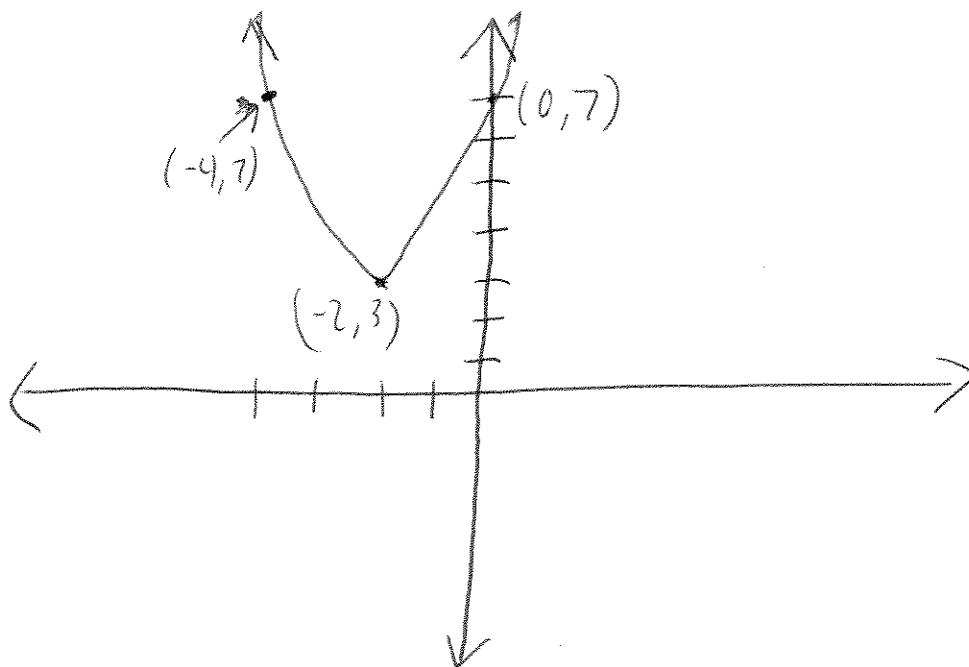
$$y = x^2 + 4x + 7$$

and sketch the graph of the parabola.

Vertex:  $x = -\frac{4}{2(1)} = -2$

$$y = (-2)^2 + (4)(-2) + 7 = 4 - 8 + 7 = 3$$

Vertex  $(-2, 3)$



18 **Logarithms** - Compute the following logarithms:

- $\log_3(27)$

$$\begin{aligned}\log_3(27) &= \log_3(3^3) \\ &= 3 \log_3(3) = \boxed{3}\end{aligned}$$

- $\log_2(0.5)$

$$\begin{aligned}&= \log_2\left(\frac{1}{2}\right) = \log_2(1) - \log_2(2) \\ &= 0 - 1 = \boxed{-1}\end{aligned}$$

- $\log_{64}(32)$  (Hint: Use the change of base formula.)

$$= \frac{\log_2(32)}{\log_2(64)} = \frac{\log_2(2^5)}{\log_2(2^6)} = \boxed{\frac{5}{6}}$$

- 19 **Compound Interest** - You invest money at an annual interest rate of 4 percent. How long (in years) will it take for your investment to quadruple (increase to four times its original amount)? Give your answer as a logarithmic expression, don't worry about a decimal value.

$$A = P(1.04)^t$$

$$A = 4P$$

$$\Rightarrow 4P = P(1.04)^t$$

$$\Rightarrow 4 = (1.04)^t$$

$$\Rightarrow \ln 4 = t \ln(1.04)$$

$$\Rightarrow \boxed{\frac{\ln 4}{\ln(1.04)} = t}$$

Note: Instead of  $\ln$  we could use  $\log_a$  for any  $a > 0, a \neq 1$ .

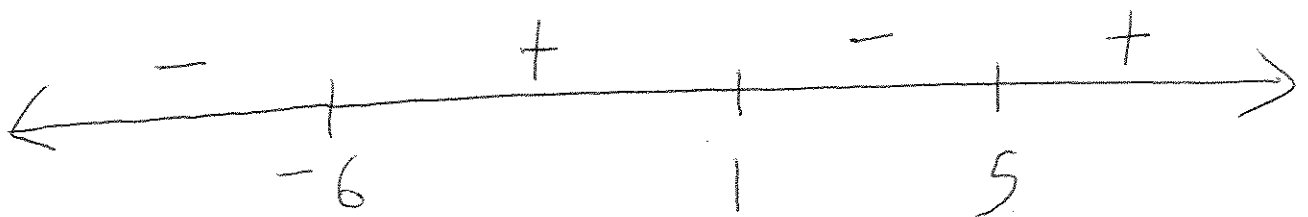
20 **Rational Inequalities** - Find the values of  $x$  for which the inequality below is true:

$$\frac{x^2 + x - 30}{x - 1} \geq 0.$$

$$x^2 + x - 30 = (x + 6)(x - 5)$$

Top is 0 for  $x = -6, 5$

Bottom is 0 for  $x = 1$



$$\frac{(-7)^2 + (-7) - 30}{(-7) - 1} = \frac{49 - 37}{-8} = \frac{12}{-8} = -\frac{3}{2} < 0$$

$$\frac{0^2 + 0 - 30}{0 - 1} = \frac{-30}{-1} = 30 > 0$$

Note:  
x cannot  
be 1.

So, ↓

$$\frac{2^2 + 2 - 30}{2 - 1} = \frac{-24}{1} = -24 < 0$$

$$\frac{6^2 + 6 - 30}{6 - 1} = \frac{12}{5} > 0.$$

$$-6 \leq x < 1$$

and

$$x \geq 5$$