Math 1010 - Exam 2

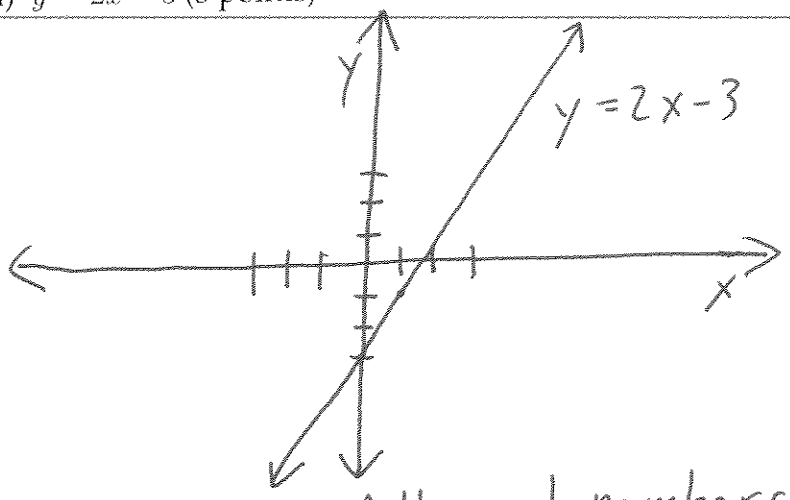
University of Utah

Fall 2009

Name: Solutions

1. Graph the following functions, and state the domain and range of each..

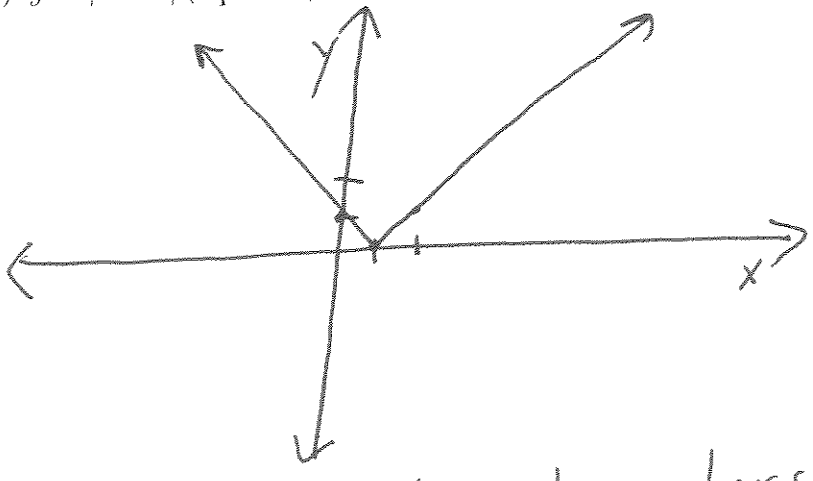
(a) $y = 2x - 3$ (3 points)



Domain (2 points): All real numbers.

Range (2 points): All real numbers.

(b) $y = |x - 1|$ (4 points)

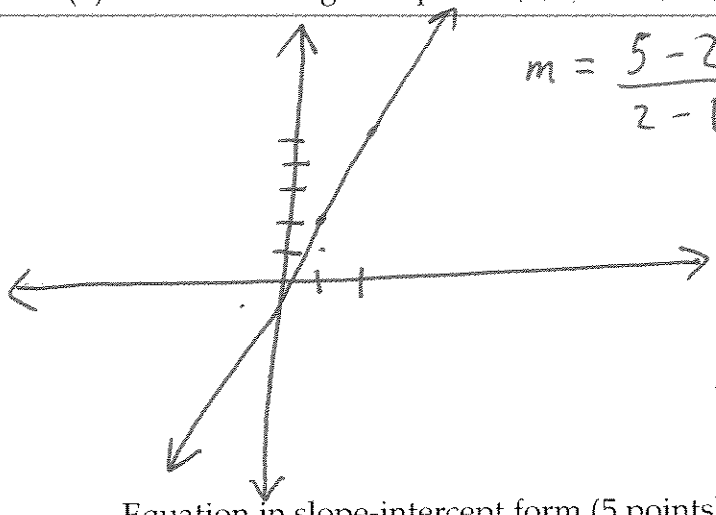


Domain (2 points): All real numbers

Range (2 points): $[0, \infty)$

2. Graph the following lines, and give their equations in *slope-intercept form*.

(a) The line through the points $(1, 2)$ and $(2, 5)$. (5 points)



$m = \frac{5-2}{2-1} = 3$

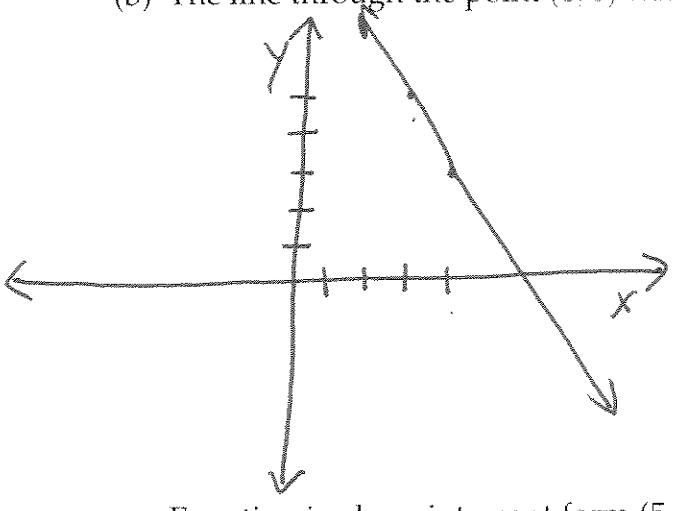
$2 = 3(1) + b$

$\Rightarrow b = -1$

$y = 3x - 1$

Equation in slope-intercept form (5 points): $\boxed{y = 3x - 1}$

(b) The line through the point $(3, 5)$ with slope $m = -2$. (5 points)



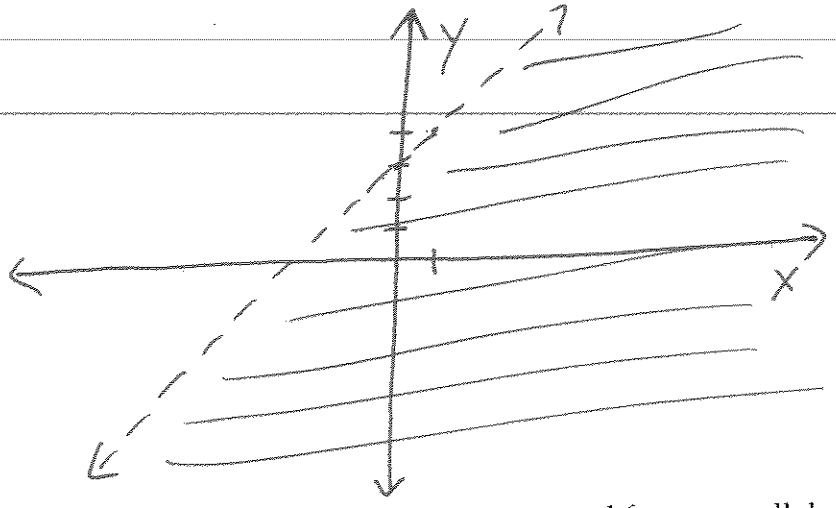
$(y-5) = -2(x-3)$

$\Rightarrow y - 5 = -2x + 6$

$\Rightarrow y = -2x + 11$

Equation in slope-intercept form (5 points): $\boxed{y = -2x + 11}$

3. Graph the inequality $y < x + 3$. (5 points)



4. Are the following lines, given in general form, parallel, perpendicular, or neither. Explain why. (5 points)

$$\begin{aligned} 2x &+ 3y = 5 \\ 3x &+ 2y = -5 \end{aligned}$$

$y = -\frac{2}{3}x + \frac{5}{3}$

$y = -\frac{3}{2}x - \frac{5}{2}$

$-\frac{2}{3} \neq -\frac{3}{2}$ So, not parallel.

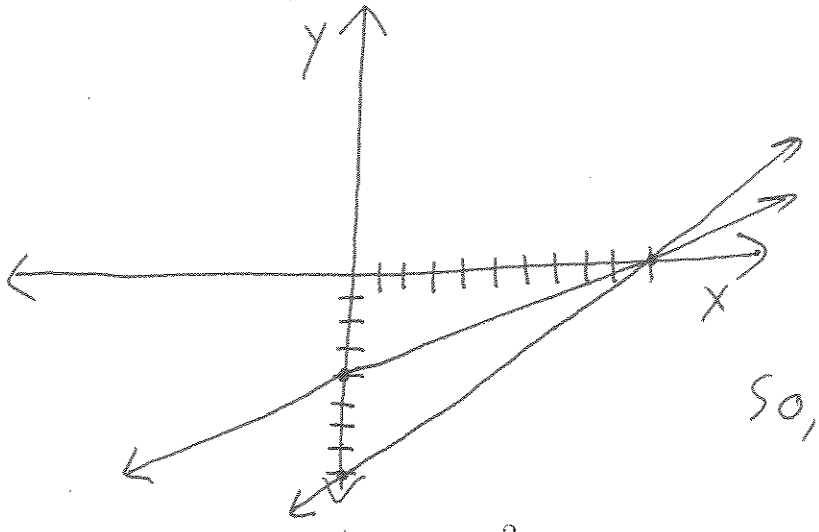
$\left(-\frac{2}{3}\right)\left(-\frac{3}{2}\right) = 1 \neq -1$ So, not perpendicular.

Neither

5. Solve the following systems of equations using any method you choose. If you use the graphical method, make sure to check and verify your answer. (6 points each)

(a) $\begin{aligned} 2x &- 5y = 20 \\ 4x &- 5y = 40 \end{aligned}$

Graphical:



Looks like they cross at (10, 0).

Check:

$2(10) - 5(0) = 20$ ✓

$4(10) - 5(0) = 40$ ✓

So, $\boxed{\begin{aligned} x &= 10 \\ y &= 0 \end{aligned}}$ a.k.a. (10, 0).

(b) $\begin{aligned} x &+ y = 2 \\ x &- 4y = 12 \end{aligned}$

Substitution:

$x = 2 - y$

$\Rightarrow (2-y) - 4y = 12 \Rightarrow 2 - 5y = 12 \Rightarrow -5y = 10$

$\Rightarrow y = -2$

$x = 2 - (-2) = 4$

So, (4, -2) a.k.a.

$\boxed{\begin{aligned} x &= 4 \\ y &= -2 \end{aligned}}$

(c) $\begin{aligned} 12x &- 14y = 15 \\ 18x &- 21y = 10 \end{aligned}$

Elimination:

$$-\frac{3}{2}(12x - 14y) = -\frac{3}{2}(15)$$

$$\Rightarrow -18x + 21y = -\frac{45}{2}$$

$$+ \quad 18x - 21y = 10$$

$$\Rightarrow \quad 0 = -\frac{25}{2}x$$

False. So, **No Solutions.**

(d) $\begin{aligned} 4x &- 3y = 25 \\ -3x &+ 8y = 10 \end{aligned}$

Elimination:

$$3(4x - 3y) = 3(25)$$

$$4(-3x + 8y) = 4(10)$$

$$\Rightarrow 12x - 9y = 75$$

$$-12x + 32y = 40$$

$$\Rightarrow \frac{23y}{23} = \frac{115}{23} \Rightarrow y = 5$$

$$4x - 3(5) = 25$$

$$\Rightarrow 4x = 40$$

$$\Rightarrow x = 10.$$

So, (10, 5) a.k.a.

$$x = 10$$
$$y = 5$$

6. Write the following numbers in scientific notation.

(a) 0.0000000381. (3 points)

3.81×10^{-8}

(b) $139,500,000$. (3 points)

1.395×10^{8}

(c) $(6.5 \times 10^{6})(2 \times 10^{4})$. (4 points)

$6.5 \times 2 = 13 \qquad 6 + 4 = 10$

$\Rightarrow \quad 13 \times 10^{10} = 1.3 \times 10^{11}$

7. Perform the following calculations on polynomials.

(a) $(4x^3 - 2x^2 + 7x + 5) + (3x^2 + 8)$. (3 points)

$4x^3 + x^2 + 7x + 13$

(b) $4x(3x^2 - 2x + 8)$. (4 points)

$12x^3 - 8x^2 + 32x$

(c) $(2x - 7)(3x^2 + 4x + 1)$. (4 points)

$(6x^3 + 8x^2 + 2x) - (21x^2 + 28x + 7)$

$= 6x^3 - 13x^2 - 26x - 7$

8. The total cost of 8 gallons of regular unleaded gasoline and 12 gallons of premium unleaded gasoline is \$71.84. Premium unleaded gasoline costs \$0.17 more per gallon than regular unleaded. Find the price per gallon for each grade of gasoline. (10 points)

x = ppg of regular

y = ppg of premium

$y - x = .17$ $\qquad x = y - .17$

$12y + 8x = \$71.84$

$\Rightarrow 12y + 8(y - .17) = 71.84$

$\Rightarrow 20y - 1.36 = 71.84$

$\Rightarrow 20y = 73.20$

$\Rightarrow y = 3.66$

$x = 3.66 - .17 = 3.49$

So, $x = \$3.49$ per gallon

$y = \$3.66$ per gallon