ASSIGNMENT 8

DYLAN ZWICK'S MATH 1010 CLASS

5.4 Factoring by Grouping and Special Forms

Write the number as a product of prime factors.

5.4.1: 6

$2 \cdot 3$

5.4.4: 12

$2 \cdot 2 \cdot 3$

5.4.5: 30

$2 \cdot 3 \cdot 5$

5.4.8: 54

$2 \cdot 3 \cdot 3 \cdot 3$

Find the greatest common factor of the expressions.

5.4.9: 16,24

8

5.4.13: x^3, x^4

x^3

5.4.15: $3x^2, 12x$

$3x$

5.4.18: $9x^3y, 24xy^2$

$3xy$

Factor out the greatest common monomial factor.(some of the polynomials have no common monomial factors.)

5.4.21: $4x + 4$

$4(x+1)$

5.4.25: $24t^2 - 36$

$12(2t^2 - 3)$

5.4.28: $y^2 - 5y$

$y(y-5)$

5.4.31: $11u^2 + 9$

No common factor other than 1

Factor a negative real number out of the polynomial and then write the polynomial factor in standard form.

5.4.41: $7 - 14x$

$-7(2x-1)$

Factor the expression by factoring out the common binomial factor.

5.4.55: $2y(y-4)+5(y-4)$

$(y-4)(2y+5)$

5.4.59: $2(7a+6)-3a^2(7a+6)$

$(7a+6)(2-3a^2)$

5.4.64: $(3x+7)(2x-1)+(x-6)(2x-1)$

$(2x-1)(4x+1)$

Factor the polynomial by grouping.

5.4.65: $x^2+25x+x+25$

$(x+25)(x+1)$

5.4.67: $y^2-6y+2y-12$

$(y-6)(y+2)$

5.4.69: x^3+2x^2+x+2

$(x+2)(x^2+1)$

5.4.71: $3a^3-12a^2-2a+8$

$(a-4)(3a^2-2)$

5.4.75: $5x^3-10x^2y+7xy^2-14y^3$

$(x-2y)(5x^2+7y^2)$

Factor the difference of two squares.

5.4.77: x^2-9

$(x+3)(x-3)$

5.4.79: $1-a^2$

$(1+a)(1-a)$

5.4.80: $16 - b^2$

$(4+b)(4-b)$

5.4.82: $9z^2 - 36$

$(3z+6)(3z-6)$

5.4.85: $4z^2 - y^2$

$(2z+y)(2z-y)$

5.4.93: $(x-1)^2 - 16$

$(x+3)(x-5)$

5.4.95: $81 - (z+5)^2$

$(14+z)(4-z)$

Factor the sum or difference of cubes.

5.4.99: $x^3 - 8$

$(x-2)(x^2+2x+4)$

5.4.103: $8t^3 - 27$

$(2t-3)(4t^2+6t+9)$

Factor the polynomial completely.

5.4.111: $8 - 50x^2$

$2(2+5x)(2-5x)$

5.4.115: $y^4 - 81$

$(y-3)(y+3)(y^2+9)$

5.4.136: *Chemical Reaction* The rate of change of a chemical reaction is given by $kQx - kx^2$, where Q is the amount of the original substance, x is the amount of substance formed, and k is a constant of the proportionality. Factor this expression.

$kx(Q-x)$

5.4.138: *Farming* A farmer has enough fencing to construct a rectanglular pig pen that encloses an area given by $32w - w^2$, where w is the width(in feet) of the pen. Use factoring to find the length of the pen in terms of w.

$32w - w^2 = (32-w)\cdot w$

the length of the pen is $32-w$

5.5 Factoring Trinomials

Factor the perfect square trinomial.

5.5.1: $x^2 + 4x + 4$

$(x+2)^2$

5.5.5: $25y^2 - 10y + 1$

$(5y-1)^2$

5.5.10: $x^2 - 14xy + 49y^2$

$(x-7y)^2$

5.5.13: $5x^2 + 30x + 45$

$5(x+3)^2$

Find two real numbers b, or one real number c such that the expressions is a perfect square trinomial.

5.5.21: $x^2 + bx + 81$

± 18

5.5.25: $x^2 + 8x + c$

16

5.5.28: $z^2 - 20z + c$

100

Factor the trinomial.

5.5.37: $x^2 + 6x + 5$

$(x+1)(x+5)$

5.5.38: $x^2 + 7x + 10$

$(x+2)(x+5)$

5.5.40: $x^2 - 10x + 24$

$(x-4)(x-6)$

5.5.41: $y^2 + 7y - 30$

$(y+10)(y-3)$

5.5.42: $m^2 - 3m - 10$

$(m-5)(m+2)$

5.5.44: $x^2 + 4x - 12$

$(x+6)(x-2)$

5.5.45: $x^2 - 20x + 96$

$(x-8)(x-12)$

5.5.49: $x^2 + 30xy + 216y^2$

$(x+12y)(x+18y)$

Factor the trinomial, if possible.(Note: Some of the trinomials may be prime.)

5.5.67: $6x^2 - 5x - 25$

$(3x+5)(2x-5)$

5.5.69: $10y^2 - 7y - 12$

$(5y+4)(2y-3)$

5.5.70: $6x^2 - x - 15$

$(3x-5)(2x+3)$

5.5.75: $2t^2 - 7t - 4$

$(2t+1)(t-4)$

5.5.77: $6b^2 + 19b - 7$

$(3b-1)(2b+7)$

5.5.79: $-2x^2 - x + 6$

$-(2x-3)(x+2)$

5.5.85: $4w^2 - 3w + 8$

Prime

5.5.87: $60y^3 + 35y^2 - 50y$

$5y(3y-2)(4y+5)$

Factor the trinomial by grouping.

5.5.93: $3x^2 + 10x + 8$

$(3x+4)(x+2)$

5.5.96: $7x^2 - 13x - 2$

$(7x+1)(x-2)$

Factor the expression completely.

5.5.99: $3x^3 - 3x$

$3x(x+1)(x-1)$

5.5.102: $16z^3 - 56z^2 + 49z$

$z(4z-7)^2$

5.5.132: *Number Problem* Let n be an integer.

(a) Factor $8n^3 + 12n^2 - 2n - 3$ so as to verify that it represents the product of three consecutive odd integers.

$(8n^3+12n^2-2n-3) = (2n-1)(2n+1)(2n+3)$

(b) If $n = 15$, what are the three integers?

29, 31, 33;

6.1 Rational Expressions and Functions

Find the domain of the rational function.

6.1.1: $f(x) = \dfrac{x^2+9}{4}$

$(-\infty, \infty)$

6.1.3: $f(x) = \dfrac{4}{x-3}$

$(-\infty, 3) \cup (3, \infty)$

6.1.4: $g(x) = \dfrac{-2}{x-7}$

$(-\infty, 7) \cup (7, \infty)$

6.1.10: $h(x) = \dfrac{4x}{x^2+16}$

$(-\infty, \infty)$

6.1.15: $f(t) = \dfrac{5t}{t^2-16}$

$(-\infty, -4) \cup (-4, 4) \cup (4, \infty)$

6.1.17: $g(y) = \dfrac{y+5}{y^2-3y}$

$(-\infty, 0) \cup (0, 3) \cup (3, \infty)$

Evaluate the rational function as indicated, and simplify.If not possible, state the reason.

6.1.23: $f(x) = \dfrac{4x}{x+3}$

(a) $f(1)$ 1

(b) $f(-2)$ -8

(c) $f(-3)$ Undefined (division by 0)

(d) $f(0)$ 0

6.1.27: $h(s) = \dfrac{s^2}{s^2-s-2}$

(a) $h(10)$ $\frac{25}{22}$

(b) $h(0)$ 0

(c) $h(-1)$ undefined

(d) $h(2)$ undefined

Describe the domain.

6.1.30: *Cost* The cost C in millions of dollars for the goverment to seize $p\%$ of an illegal drug as it enters the country is given by

$$C = \frac{528p}{100-p}.$$

$[0, 100)$

6.1.31: *Inventory Cost* The inventory cost I when x units of a product are ordered from a supplier is given by

$$I = \frac{0.25x + 200}{x}.$$

$\{1, 2, 3, 4, \ldots\}$

Simplify the rational expression.

6.1.43: $\frac{5x}{25}$

$\frac{x}{5}$

6.1.45: $\frac{12x^2}{12x}$

$x, x \neq 0$

6.1.51: $\frac{x^2(x-8)}{x(x-8)}$

$x, x \neq 8, x \neq 0$

6.1.52: $\frac{a^2b(b-3)}{b^3(b-3)^2}$

$\frac{a^2}{b^2(b-3)}$

6.1.55: $\frac{y^2-49}{2y-14}$

$\frac{y+7}{2}, y \neq 7$

6.1.58: $\frac{u^2-12u+36}{u-6}$

$u-6, u \neq 6$

6.1.60: $\frac{z^2+22z+121}{3z+33}$

$\frac{z+11}{3}, z \neq -11$

6.1.65: $\frac{3x^2-7x-20}{12+x-x^2}$

$-\left(\frac{3x+5}{x+3}\right), x \neq 4$

6.1.71: $\frac{3xy^2}{xy^2+x}$

$\frac{3y^2}{y^2+1}, x \neq 0$

6.1.73: $\frac{y^2-64x^2}{5(3y+24x)}$

$\frac{y-8x}{15}, y \neq -8x$

6.1.75: $\frac{5xy+3x^2y^2}{xy^3}$

$\frac{5+3xy}{y^2}, x \neq 0$

6.1.78: $\dfrac{x^2+4xy}{x^2-16y^2}$

$\dfrac{x}{x-4y}$; $x \neq -4y$

6.1.87: *Average Cost* A machine shop has a setup cost of \$2500 for the production of a new product. The cost of labor and material for producing each unit is \$9.25.

(a) Write the total cost C as a function of x,the number of units produced.

$C = 2500 + 9.25x$

(b) Write the average cost per unit $\overline{C} = C/x$ as a function of x, the number of units produced.

$\overline{C} = \dfrac{C}{x} = \dfrac{2500 + 9.25x}{x}$

(c) Determine the domain of the function in part (b).

$\{1, 2, 3, 4, \cdots\}$

6.1.88: *Average Cost* A greeting card company has an initial investement of \$60,000. The cost of producing one dozen card is \$6.50.

(a) Write the total cost C as a function of x, the number of dozens of cards produced.

$C = 60{,}000 + 6.50x$

(b) Write the average cost per dozen $\overline{C} = C/x$ as a function of x, the number of dozens of cards produced.

$\overline{C} = \dfrac{60{,}000 + 6.50x}{x}$

(c) Determine the domain of the funciton in part (b).

$\{1, 2, 3, 4\}$

(d) \$11.95