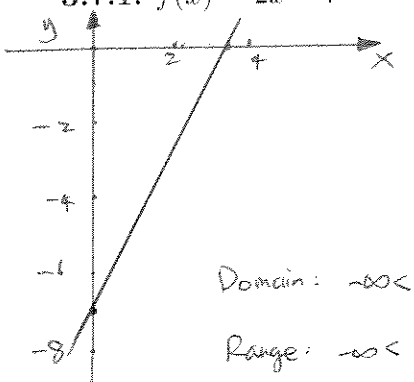
ASSIGNMENT 6

DYLAN ZWICK'S MATH 1010 CLASS

3.7 Graphs of Functions

Sketch the graph of the function. Then determine its domain and range.

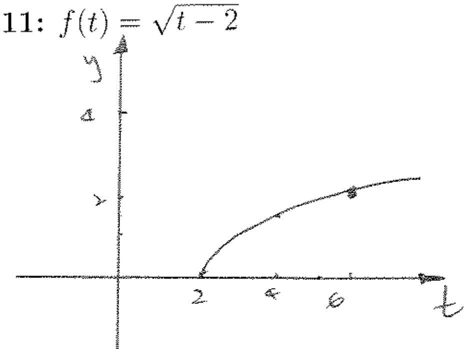
3.7.1: $f(x) = 2x - 7$



Domain: $-\infty < x < \infty$

Range: $-\infty < y < \infty$

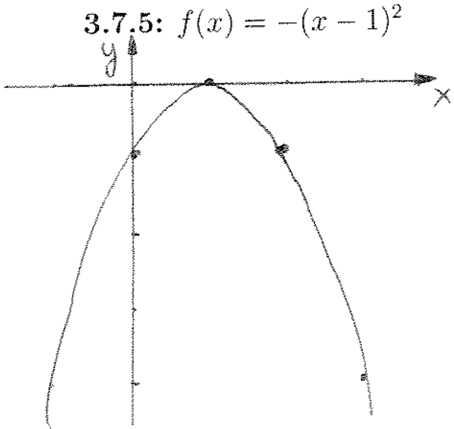
3.7.11: $f(t) = \sqrt{t-2}$



DOMAIN: $2 \le t < \infty$

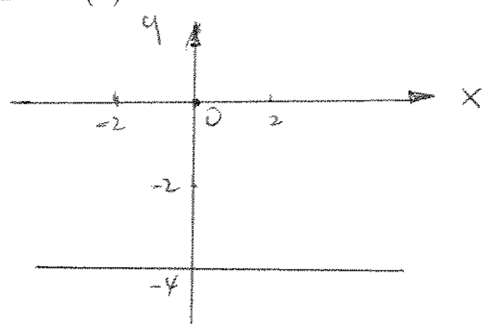
Range: $0 \le y < \infty$

3.7.5: $f(x) = -(x-1)^2$



DOMAIN: $-\infty < x < \infty$ RANGE: $-\infty < y \le 0$

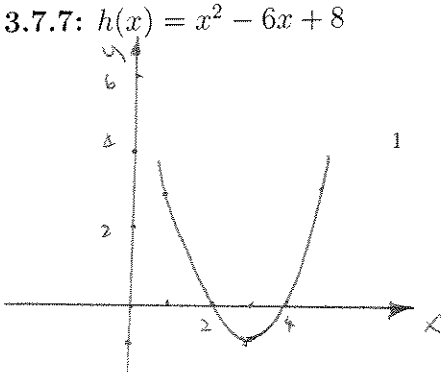
3.7.14: $H(x) = -4$



Domain: $-\infty < x < \infty$

Range: $y = -4$

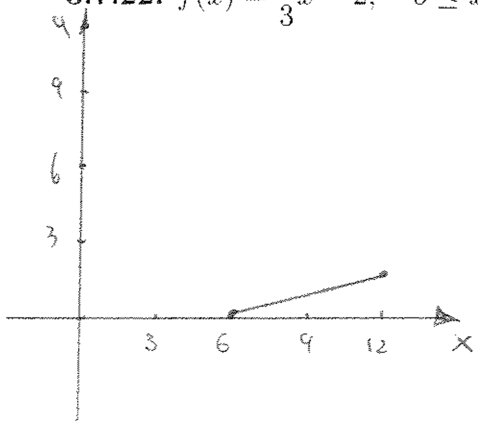
3.7.7: $h(x) = x^2 - 6x + 8$



Domain ~~Range~~: $-\infty < x < \infty$

Range: $-1 \le y < \infty$

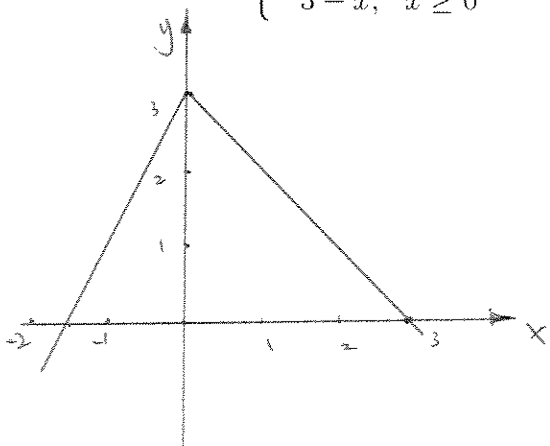
3.7.22: $f(x) = \frac{1}{3}x - 2, \quad 6 \le x \le 12$



DOMAIN: $6 \le x \le 12$

RANGE: $0 \le y \le 2$

3.7.25: $h(x) = \begin{cases} 2x+3, & x < 0 \\ 3-x, & x \ge 0 \end{cases}$

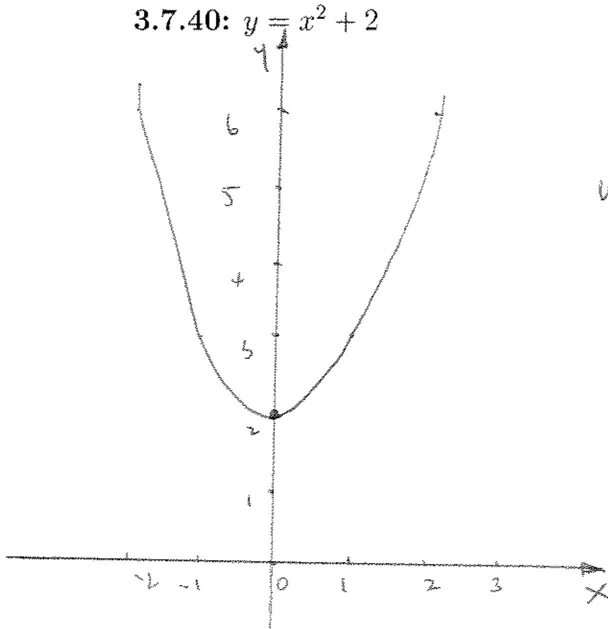


DOMAIN: $-\infty < x < \infty$

RANGE: $-\infty < y \le 3$

Sketch a graph of the equation.Use the Vertical Line Test to determine whether y is function of x.

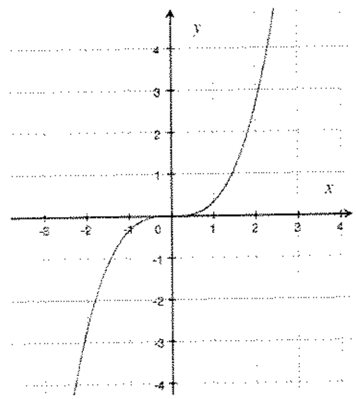
3.7.40: $y = x^2 + 2$



y is a function of x.

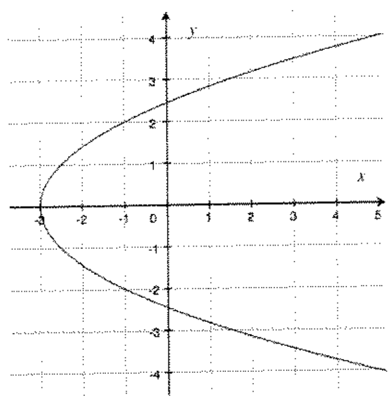
Use the Vertical Line Test to determine whether y is a function of x.

3.7.33: $y = \frac{1}{3}x^3$



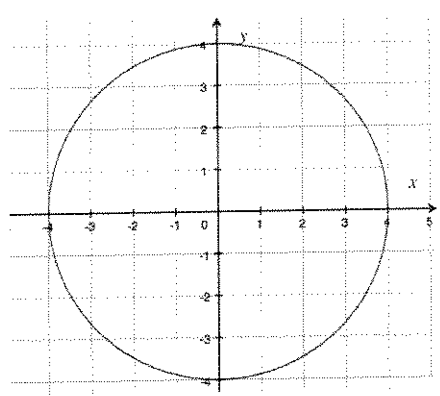
Yes, $y = \frac{1}{3}x^3$ passes the vertical Line Test and is a function of x.

3.7.36: $-2x + y^2 = 6$



No, y is not a function of x by the Vertical Line Test.

3.7.37: $x^2 + y^2 = 16$

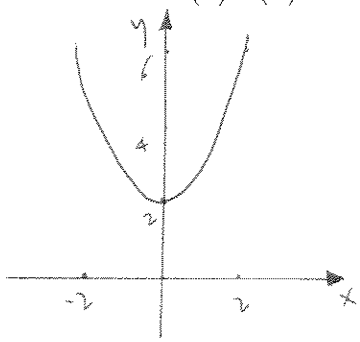


No, y is not a function of x by the Vertical Line Test.

Identify the transformation of f, and sketch a graph of the function h.

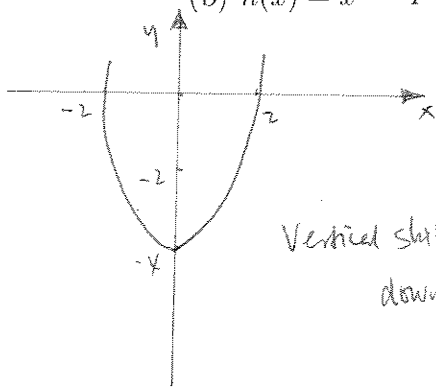
3.7.47: $f(x) = x^2$

(a) $h(x) = x^2 + 2$



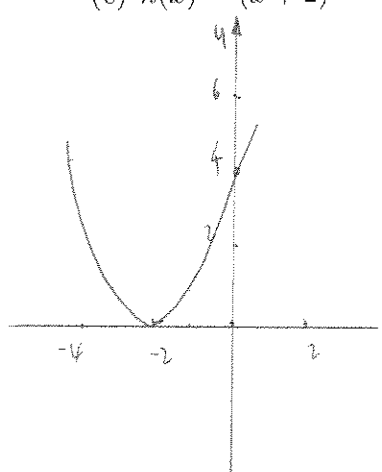
Vertical shift 2 units upward

(b) $h(x) = x^2 - 4$



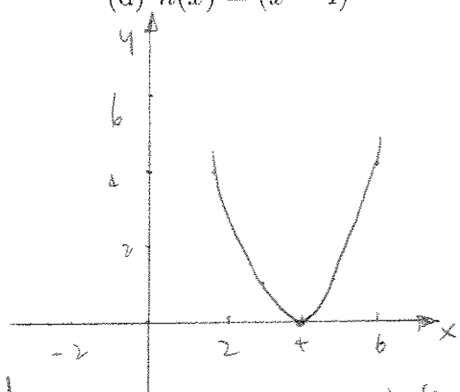
Vertical shift 4 units downward.

(c) $h(x) = (x + 2)^2$



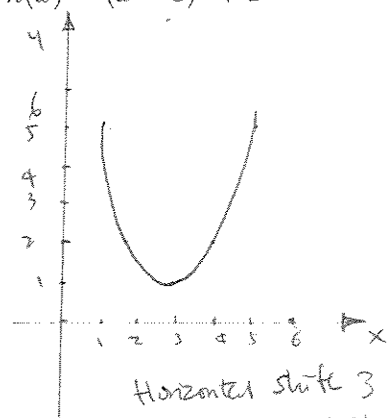
Horizontal shift 2 units to the left.

(d) $h(x) = (x - 4)^2$



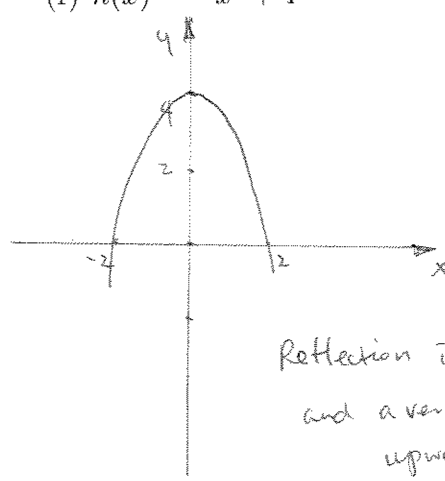
Horizontal shift 4 units to the right

(e) $h(x) = (x - 3)^2 + 1$



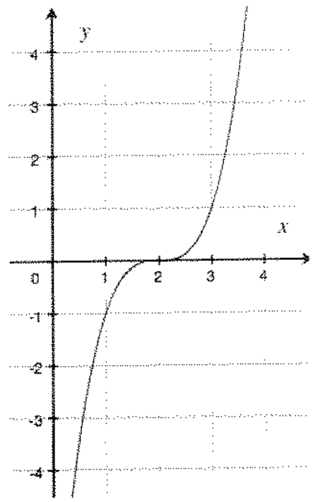
Horizontal shift 3 units to the right and a vertical shift 1 unit upward.

(f) $h(x) = -x^2 + 4$



Reflection in the x-axis and a vertical shift 4 units upward.

3.7.69: Identify the basic function and any transformation shown in the graph. Write the equation for the graphed fuction.

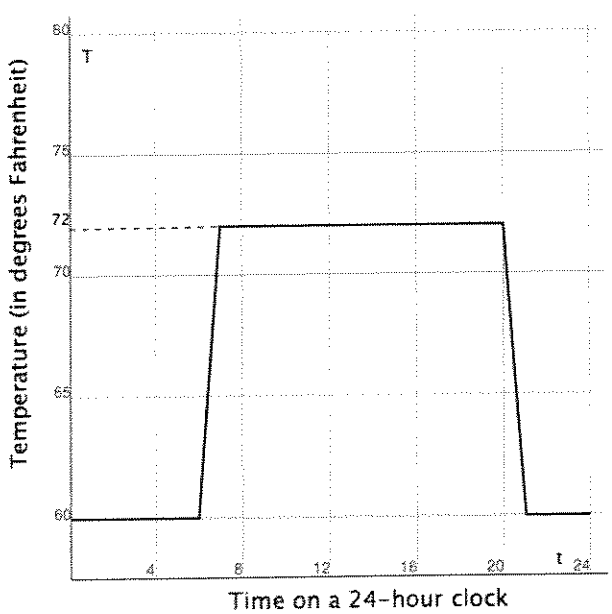


basic function : $y = x^3$

Transformation : Horizontal shift 2 units right

Equation : $y = (x-2)^3$

3.7.82: ***Graphical Reasoning*** An electronically controlled thermostat in a home is programmed to lower the temperature automatically during the night. The temperature T, in degrees Fahrenheit, is given in terms of t, the time on a 24-hour clock(see figure).



(a) Explain why T is function of t.

T is a function of t because to each time t there corresponds one and only one temperature T.

(b) Find $T(4)$ and $T(15)$.

$T(4) = 60°$

$T(15) = 72°$

(c) The thermostat is reprogrammed to produce a temperature H, where $H(t) = T(t-1)$. Explain how this changes the temperature in the house.

If the thermostat were reprogrammed to produce a temperature H where $H(t) = T(t-1)$, all the temperature changes would occur 1 hour later.

(d) The thermostat is reprogrammed to produce a temperature H, where $H(t) = T(t) - 1$. Explain how this changes the temperature in the house.

The Temperature would be decreased by 1 degree.

4.1 Systems of Equations

Determine whether each ordered pair is a solution of the system of equations.

4.1.1: $\begin{cases} x + 2y &= 9 \\ -2x + 3y &= 10 \end{cases}$

(a) $(1, 4)$

Solution

(b) $(3, -1)$

Not a Solution

4.1.2: $\begin{cases} 5x - 4y &= 34 \\ x - 2y &= 8 \end{cases}$

(a) $(0, 3)$ Not a Solution

(b) $(6, -1)$ Solution

4.1.5: $\begin{cases} 4x - 5y &= 12 \\ 3x + 2y &= -2.5 \end{cases}$

(a) $(8, 4)$

Not a Solution

(b) $(\frac{1}{2}, -2)$

Solution

State the number of solutions of the system of linear equations without solving the system.

4.1.9: $\begin{cases} y &= 4x \\ y &= 4x + 1 \end{cases}$

The equations have the same slope, So the lines are parallel, The System of linear equations has no solution.

4.1.10: $\begin{cases} y &= 3x + 2 \\ y &= -3x + 2 \end{cases}$

Different Slopes, the lines intersect at one pt. The system of linear equations has one solution.

4.1.14: $\begin{cases} y &= \frac{2}{3}x + 1 \\ 3y &= 2x + 3 \end{cases}$

The equations are the same, so the lines coincide. It has infinitely many solutions.

Determine whether the system is consistent or inconsistent.

4.1.15: $\begin{cases} x + 2y &= 6 \\ x + 2y &= 3 \end{cases}$

inconsistent

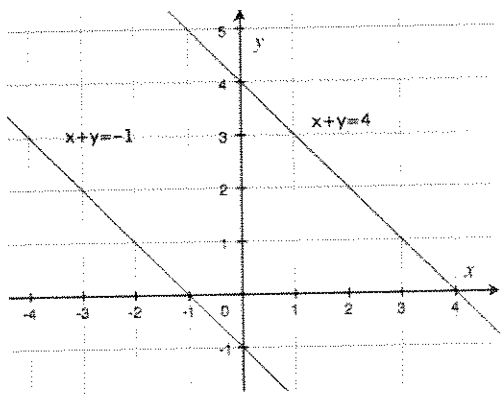
4.1.19: $\begin{cases} -x + 4y &= 7 \\ 3x - 12y &= -21 \end{cases}$

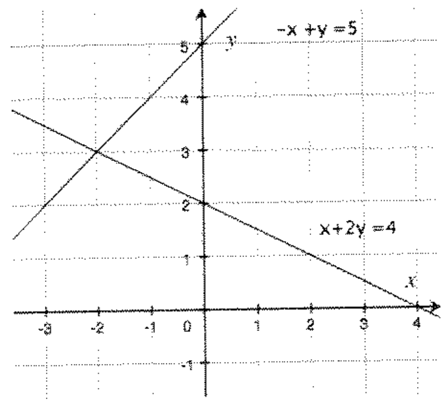
Consistent.

Use the graphs of the equations to determine whether the system has any solutions. Find any solutions that exist.

4.1.27: $\begin{cases} x+y &= 4 \\ x+y &= -1 \end{cases}$

4.1.28: $\begin{cases} -x+y &= 5 \\ x+2y &= 4 \end{cases}$



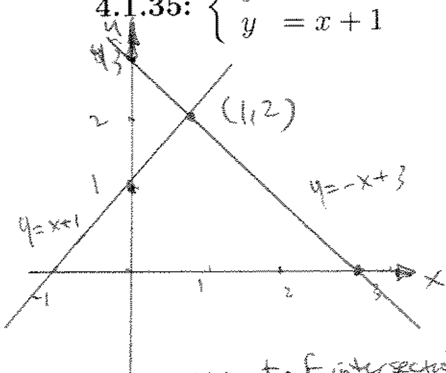


The slopes are the same, so there is no solution.

The is one solution at (-2,3)

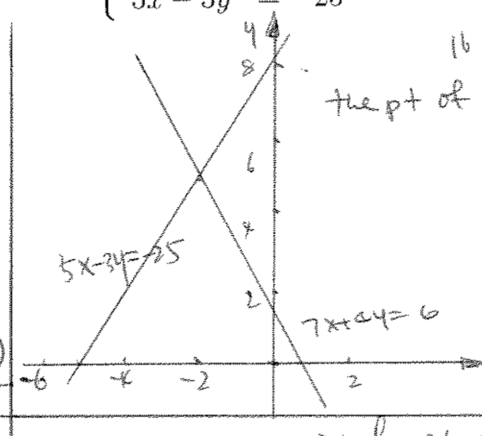
Use the graphical method to solve the system of equations.

4.1.35: $\begin{cases} y &= -x+3 \\ y &= x+1 \end{cases}$



the pt of intersection is (1,2)

4.1.48: $\begin{cases} 7x+4y &= 6 \\ 5x-3y &= -25 \end{cases}$



16

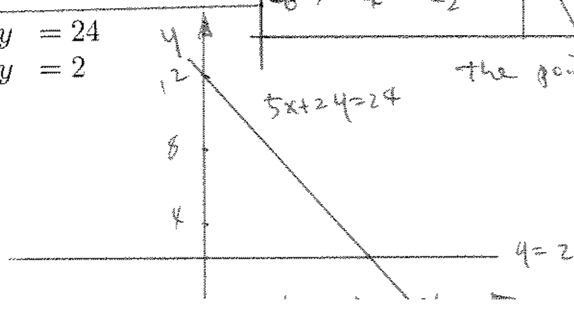
the pt of intersection is (-2,5)

4.1.40: $\begin{cases} 5x+2y &= 24 \\ y &= 2 \end{cases}$

$5x+2y=24$

$2y=-5x+24$

$y=-\frac{5}{2}x+12$



the point of intersection is (4,2)

Solve the system of equations by the method of substitution.

4.1.53: $\begin{cases} x - 2y &= 0 \\ 3x + 2y &= 8 \end{cases}$

Solve for x in the first equation

$x=2y$

Substitute into the 2nd equation

$3(2y)+2y=8$

$8y=8$ Solution is

$y=1$ $(x,y)=(2,1)$

$x=2y=2$

4.1.57: $\begin{cases} x + y &= 3 \\ 2x - y &= 0 \end{cases}$

Substitute $y=3-x$ in the 2nd equation

$2x-(3-x)=0$

$x=1$

$y=3-x=2$

Solution $(x,y)=(1,2)$

4.1.61: $\begin{cases} 3x + y &= 8 \cdots (1) \\ 3x + y &= 6 \cdots (2) \end{cases}$

(1) gives $y=8-3x \cdots (3)$

Substitute (3) into (2) gives

$3x+(8-3x)=6$

LHS $=8$ RHS $=6$

There is no solution.

4.1.66: $\begin{cases} x + 4y &= 300 \\ x - 2y &= 0 \end{cases}$

Substitute $x=2y$ into the 1st equation gives

$6y=300$

$y=50$

$x=2y=100$

The solution is $(x,y)=(100,50)$

4.1.69: $\begin{cases} 4x - 14y &= -15 \\ 18x - 12y &= 9 \end{cases}$

The 1st equation gives $x=\frac{-15+14y}{4}$

Substitute into the 2nd equation

we have $18\left(\frac{-15+14y}{4}\right)-12y=9$

$18(-15+14y)-48y=36$

$204y=306$

$y=\frac{3}{2}$

$x=\frac{-15+14y}{4}=\frac{3}{2}$

The solution is $(x,y)=(\frac{3}{2},\frac{3}{2})$

4.1.72: $\begin{cases} \frac{1}{2}x + \frac{3}{4}y &= 10 \cdots (1) \\ \frac{3}{2}x - y &= 4 \cdots (2) \end{cases}$

(2) gives $y=\frac{3}{2}x-4$

Substitute into (1)

gives $\frac{1}{2}x+\frac{3}{4}(\frac{3}{2}x-4)=10$

$x=8$

$y=\frac{3}{2}x-4=\frac{3}{2}\times 8-4=8$

Solution is $(8,8)$

4.1.95: ***Hay Mixture*** A farmer wants to mix two types of hay. The first type sells for \$ 125 per ton and the second type sells for \$75 per ton. The farmer wants a total of 100 tons of hay at a cost of \$90 per ton. How many tons of each type of hay should be used in the mixture?

Let x, y be the amount of the first type and the 2nd type of hay respectively

Then $x+y=100 \cdots (1)$

$125x+75y=90(100) \cdots (2)$

(1) gives $x=100-y$ Substitute into (2) gives $125(100-y)+75y=90(100)$

4.1.97: ***Break-Even Analysis*** A small business invests \$8000 in equipment to produce a new candy bar. Each bar costs \$1.20 to produce and is sold for \$2.00. How many candy bars must be sold before the business breaks even?

Let x be the number of bars sold, then the total cost is $8000 + 1.20x$, and the total revenue is $2.00x$. Break even points occurs when

$$8000 + 1.20x = 2.00x \Rightarrow x = 10{,}000$$

10,000 candy bars must be sold.

4.1.101: ***Investment*** A total of \$ 12,000 is invested in two bonds that pay 8.5% and 10% simple interest. The annual interest is \$1140. How much is invested in each bond?

let x and y be the number of the 8.5%, 10% bond respectively.

Then $x + y = 12{,}000$

$0.085x + 0.10y = 1140$

Solving the system gives $x = 4000$, $y = 8000$

So \$4000 is at 8.5% and \$8000 is at 10%

Number Problems Find two positive integers that satisfy the given requirements.

4.1.103: The sum of the two numbers is 80 and their difference is 18.

Let x and y be the two numbers, with x being the large.

Then $x + y = 80$
$x - y = 18$, $\Rightarrow$ $x = 49$ $y = 31$

The numbers are 49 and 31.

4.1.110: The difference of the numbers is 86 and the larger number is three times the smaller number.

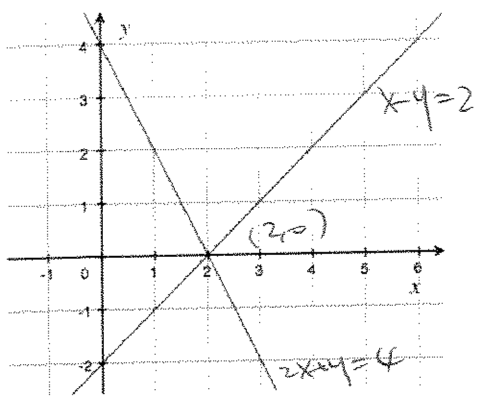
let x be the larger number, y the smaller number.

Then $x = 3y$
$x - y = 86$ $\Rightarrow$ $x = 129$ $y = 43$.

4.2 Linear Systems in Two Variables

Solve the system of linear equations by the method of elimination. Identify and label each line with its equation, and label the point of intersection(if any).

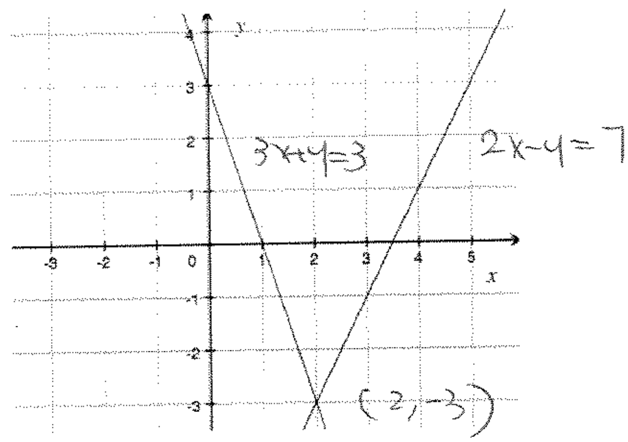
4.2.1: $\begin{cases} 2x + y &= 4 \\ x - y &= 2 \end{cases}$



$2x + y = 4$
$x - y = 2$
$3x = 6$
$x = 2$

$2 - y = 2$
$-y = 0$
$y = 0$

4.2.5: $\begin{cases} 3x + y &= 3 \\ 2x - y &= 7 \end{cases}$



$3x + y = 3$
$2x - y = 7$
$5x = 10$
$x = 2$
$2 \cdot 2 - y = 7$
$y = -3$

Solve the system of linear equations by the method of elimination.

4.2.13: $\begin{cases} 6x - 6y &= 25 \\ 3y &= 11 \end{cases}$

$6x - 6y = 25$
$6y = 22$
$6x = 47$

$x = \frac{47}{6}$

$3y = 11$

$y = \frac{11}{3}$

Solution $\left(\frac{47}{6}, \frac{11}{3}\right)$

4.2.19: $\begin{cases} 5x + 2y = 7 \\ 3x - y = 13 \end{cases}$

$5x + 2y = 7$
$6x - 2y = 26$
$11x = 33$
$x = 3$

$3 \cdot 3 - y = 13$
$y = -4$

Solution: $(3, -4)$

4.2.21: $\begin{cases} x - 3y &= 2 \\ 3x - 7y &= 4 \end{cases}$

$-3x + 9y = -6$

$3x - 7y = 4$

$2y = -2$

$y = -1$

$x - 3(-1) = 2$

$x = -1$

Solution $(-1,-1)$

4.2.25: $\begin{cases} 2u + 3v &= 8 \\ 3u + 4v &= 13 \end{cases}$

$-6u - 9v = -24$

$6u + 8v = 26$

$-v = 2$

$v = -2$

$2u + 3(-2) = 8$

$u = 7$

Solution $(7,-2)$

4.2.27: $\begin{cases} 12x - 5y &= 2 \\ -24x + 10y &= 6 \end{cases}$

$24x - 10y = 4$

$-24x + 10y = 6$

$0 \neq 10$

No solution

4.2.35: $\begin{cases} 5x + 7y &= 25 \\ x + 1.4y &= 5 \end{cases}$

$5x + 7y = 25 \Rightarrow 5x + 7y = 25$

$x + 1.4y = 5 \Rightarrow -5x - 7y = -25$

$0 = 0$

infinitely many solution

4.2.37: $\begin{cases} \frac{1}{2}x - \frac{1}{3}y &= 1 \\ \frac{1}{4}x - \frac{1}{9}y &= \frac{2}{3} \end{cases}$

$\frac{1}{2}x - \frac{1}{3}y = 1 \Rightarrow -6x + 4y = -12$

$\frac{1}{4}x - \frac{1}{9}y = \frac{2}{3} \Rightarrow 9x - 4y = 24$

$3x = 12$

$x = 4$

$\cancel{3x = 24} \quad \frac{1}{2}4 - \frac{1}{3}y = 1$

$\Rightarrow y = 3$

Solution $(4,3)$

Solve the system of linear equations by any convenient method.

4.2.41: $\begin{cases} x+7y &= -6 \\ x-5y &= 18 \end{cases}$

$x+7y=-6$
$-x+5y=-18$
$12y=-24$
$y=-2$
$x=5y+18=8$

The Solution is $(8,-2)$

4.2.45: $\begin{cases} 2x-y &= 20 \\ -x+y &= -5 \end{cases}$

$2x-y=20$
$-x+y=-5$
$x=15$
$-15+y=-5$
$y=10$

Solution: $(15,10)$

Decide whether the system is consistent or inconsistent.

4.2.49: $\begin{cases} 4x-5y &= 3 \\ -8x+10y &= -6 \end{cases}$

consistent

4.2.61: ***Break-Even Analysis*** To open a small business, you need an initial investment of \$85,000. Your costs each week will be about \$7400. Your projected weekly revenue is \$8300. How many weeks will it take to break even?

Let x be the number of weeks to break even.

The the total revenue and costs are

$8300x$ and $85{,}000 + 7400x$ resp.

$8300x = 85{,}000 + 7400x$

$x \approx 94.44$

It will take 95 weeks to break even.

4.2.63: ***Comparing costs*** A band charges \$500 to play for 4 hours plus \$50 for each additional hour. A DJ costs \$300 to play for 4 hours plus \$75 for each additional hour. After how many hours will the cost of the DJ exceed the cost of the band?

Let x be the number of hours.

cost for band: $500 + 50x$

cost for DJ: $300 + 75x$

let $500 + 50x = 300 + 75x$

$\Rightarrow x = 8$

So after ~~8 hours~~ 8 additional hours will the cost for DJ exceed the cost of a band.

4.2.67: ***Average Speed*** A van travels for 2 hours at an average speed of 40 miles per hour. How much longer must the van travel at an average speed of 55 miles per hour so that the average speed for the total trip will be 50 miles per hour?

Suppose the van needs to run x hour,

then $40 \cdot 2 + x \cdot 55 = 50(x+2)$

$\Rightarrow x = 4$

So the van must travel 4 hours longer.

4.2.71: ***Ticket Sales*** Five hundred tickets were sold for a fundraising dinner. The receipts totaled \$3400.00. Adult tickets were\$7.50 each and children's tickets were \$4.00 each. How many tickets of each type were sold?

let the number for adult tickets sale and children tickets sale be x and y

then
$$\left.\begin{aligned} x + y &= 500 \\ 7.5x + 4.0y &= 3400 \end{aligned}\right\} \Rightarrow \begin{aligned} x &= 400 \\ y &= 100 \end{aligned}$$

There were 400 adult tickets and 100 children tickets sold.

4.2.80: ***Acid Mixture*** Fifty gallons of a 60% acid solution is obtained by mixing an 80% solution with a 50% solution. How many gallons of each solution must be used to obtain the desired mixture?

Amount of 80% solution $= x$

Amount of 50% solution $= y$

$$\begin{cases} x+y=50 \\ 0.8x+0.5y=50\cdot 0.6 \end{cases} \Rightarrow x=\frac{50}{3} \quad y=\frac{100}{3}$$

There are about $\frac{100}{3}$ gallon 50% solution and $\frac{50}{3}$ gallons of 80% solution.

4.2.83: ***Best-Fitting Line*** The slope and y-intercept of the line $y = mx + b$ that best fits the three noncollinear points $(0,0),(1,1),(2,3)$ are given by the solution of the following system of linear equations.

$$\begin{cases} 5m + 3b &= 7 \\ 3m + 3b &= 4 \end{cases}$$

(a) Solve the system and find the equation of the best-fitting line.

$$\begin{array}{r} 5m+3b=7 \\ -3m-3b=-4 \\ \hline 2m \quad = 3 \end{array}$$

$m=\frac{3}{2}$

$5(\frac{3}{2})+3b=7 \quad b=-\frac{1}{6}$

The line is

$y=\frac{3}{2}x-\frac{1}{6}$

(b) Plot the three points and sketch the graph of the best fitting line.