ASSIGNMENT 14

DYLAN ZWICK'S MATH 1010 CLASS

9.1 Exponential Functions

Simplify the expression.

9.1.1: $3^x \cdot 3^{x+2}$

3^{2x+2}

9.1.4: $\dfrac{3^{2x+3}}{3^{x+1}}$

3^{x+2}

9.1.3: $\frac{e^{x+2}}{e^x}$

e^2

9.1.5: $3(e^x)^{-2}$

$\dfrac{3}{e^{2x}}$

Evaluate the function as indicated. Use a calculator ONLY IF it is necessary or more efficient.

9.1.17: $f(x) = 3^x, (a)x = -2, (b)x = 0, (c)x = 1.$

(a) $\frac{1}{9}$, (b) 1, (c) 3

9.1.21: $f(t) = 500(\frac{1}{2})^t, (a)t = 0, (b)t = 1, (c)t = \pi.$

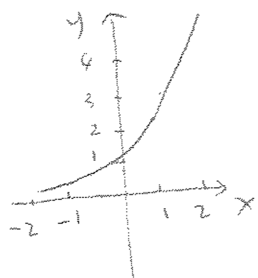
(a) 500, (b) 250, (c) 56.657

9.1.26: $P(t) = \dfrac{10,000}{(1.01)^{12t}}, (a)t = 2, (b)t = 10, (c)t = 20$

(a) 7875.661 (b) 3029.948 (c) 918.058

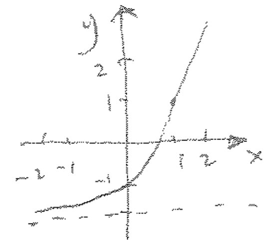
Sketch the graph of the function. Identify the horizontal asymptote.

9.1.31: $f(x) = 3^x$



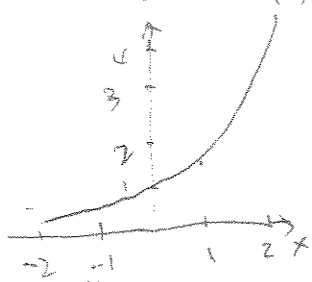
Horizontal asymptote: $y=0$

9.1.35: $g(x) = 3^x - 2$



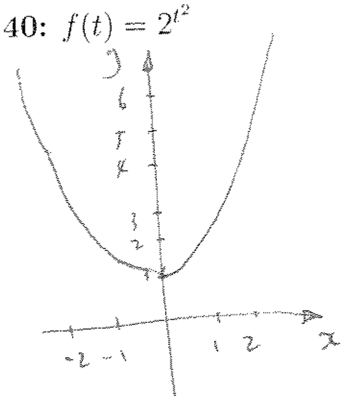
Horizontal asymptote: $y=-2$

9.1.32: $h(x) = \frac{1}{2}(3^x)$



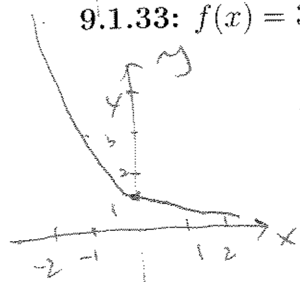
Horizontal asymptote: $y=0$

9.1.40: $f(t) = 2^{t^2}$



No horizontal asymptote.

9.1.33: $f(x) = 3^{-x}$



Horizontal asymptote: $y=0$

9.1.69: *Radioactive Decay* After t years, 16 grams of a radioactive element with a half-life of 30 years decays to a mass y(in grams) given by $y = 16(\frac{1}{2})^{t/30}, t \geq 0$. How much of the initial mass remains after 80 years?

$$y = 16\left(\frac{1}{2}\right)^{80/30} \approx 2.520 \text{ grams}$$

9.1.70: *Radioactive Substance* In July of 1999, an individual bought several leaded containers from a metals recycler and found two of them labeled"radioactive". An investigation showed that the containers, originally obtained from Ohio State University, apparently had been used to store iodine-131 starting in January of 1999.Because iodine-131 has a half life of only 8 days, no elevated radiation levels were detected. Suppose 6 grams of iodine-131 is stored in January. The mass y (in grams) that remains after t days is given by $y = 6(\frac{1}{2})^{t/8}, t \geq 0$. How much of the substance is left in July, after 180 days have passed.

$$y = 6\left(\frac{1}{2}\right)^{180/8} \approx 1.01 \times 10^{-6} = 0.00000101 \text{ gram}$$

9.1.71: *Compound Interest* A sum of \$5000 is invested at an annual interest rate of 6%, compounded monthly. Find the balance in the account after 5 years.

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$A = 5000\left(1 + \frac{0.06}{12}\right)^{12(5)} \approx \$6744.25$$

9.1.72: *Compound Interest* A sum of \$2000 is invested at an annual interest rate of 8%, compounded quarterly. Find the balance in the account after 10 years.

$$A = 2000\left(1 + \frac{0.08}{4}\right)^{4(10)}$$

$$A \approx \$4416.08$$

Compound Interest Complete the table to determine the balance A for P dollars invested at rate r for t years, compounded n times per year.

9.1.73: $P = \$100, r = 7\%, t = 15$ years.

n	1	4	12	365	Continuous compounding
A					

275.90 ; 283.18 ; 284.89 ; 285.74 ; 285.77

9.1.75: $P = \$2000, r = 9.5\%, t = 10$ years.

n	1	4	12	365	Continuous compounding
A					

4956.46 ; 5114.30 ; 5152.~~0~~11 ; 5170.78 ; \$171.42

Compounded continuously : $A = Pe^{rt} = 2000 e^{0.095(10)} = 5171.42$

Compound Interest Complete the table to determine the principal P that will yield a balance of A dollars when invested at rate r for t year, compounded n times per year.

9.1.77: $A = \$5000, r = 7\%, t = 10$ years

n	1	4	12	365	Continuous compounding
P					

\$2541.75 ; \$2498.00 ; \$2487.98 ; \$2483.09 ; \$2482.93

9.1.79: $A = \$1,000,000, r = 10.5\%, t = 40$ years

n	1	4	12	365	Continuous compounding
P					

\$18,429.30 ; \$15,830.43 ; \$15,272.04; \$ 15,004.64;
\$ 14995.58

9.1.82: *Population Growth* The populations P (in millions) of the United States from 1980 to 2006 can be approximated by the exponential function $P(t) = 226(1.0110)^t$, where t is the time in years, with $t = 0$ corresponding to 1980. Use the model to estimate the populations in years (a) 2010 and (b) 2020.

(a) $P(30) = 226(1.0110)^{30} \approx 313,793,000$ people

(b) $P(40) = 226(1.0110)^{40} \approx 350,070,000$ people

9.2 Composite and Inverse Functions

Find the compositions.

9.2.1: $f(x) = 2x + 3, g(x) = x - 6$

(a) $(f \circ g)(x)$

$2x-9$

(c) $(f \circ g)(4)$

-1

(b) $(g \circ f)(x)$

$2x-3$

(d) $(g \circ f)(7)$

11

9.2.2: $f(x) = x - 5, g(x) = 3x + 2$

(a) $(f \circ g)(x)$

$3x-3$

(c) $(f \circ g)(3)$

6

(b) $(g \circ f)(x)$

$3x-13$

(d) $(g \circ f)(3)$

-4

9.2.3: $f(x) = x^2 + 3, g(x) = x + 2$

(a) $(f \circ g)(x)$

$(f \circ g)(x) = (x+2)^2 + 3$
$= x^2 + 4x + 7$

(c) $(f \circ g)(2)$

19

(b) $(g \circ f)(x)$

$(g \circ f)(x) = (x^2+3) + 2$
$= x^2 + 5$

(d) $(g \circ f)(-3)$

14

9.2.5: $f(x) = |x - 3|, g(x) = 3x$

(a) $(f \circ g)(x)$

$|3x-3|$

(c) $(f \circ g)(1)$

0

(b) $(g \circ f)(x)$

$3|x-3|$

(d) $(g \circ f)(2)$

3

9.2.8: $f(x) = \sqrt{x + 6}, g(x) = 2x - 3$

(a) $(f \circ g)(x)$

$\sqrt{2x+3}$

(c) $(f \circ g)(3)$

3

(b) $(g \circ f)(x)$

$2\sqrt{x+6} - 3$

(d) $(g \circ f)(-2)$

1

9.2.10: $f(x) = \dfrac{4}{x^2 - 4}, g(x) = \dfrac{1}{x}$

(a) $(f \circ g)(x)$

$(f\circ g)(x) = \dfrac{4}{(\frac{1}{x})^2 - 4} = \dfrac{4x^2}{1-4x^2}$ $x \neq 0$

(c) $(f \circ g)(-2)$

$-\dfrac{16}{15}$

(b) $(g \circ f)(x)$

$(g\circ f)(x) = \dfrac{1}{\frac{4}{x^2-4}} = \dfrac{x^2-4}{4}$, $x = \pm 2$

(d) $(g \circ f)(1)$

$-\dfrac{3}{4}$

9.2.11: Use the functions f and g to find the indicated values.
$f = \{(-2, 3), (-1, 1), (0, 0), (1, -1), (2, -3)\}$
$g = \{(-3, 1), (-1, -2), (0, 2), (2, 2), (3, 1))\}$

(a) $f(1)$ -1

(c) $(g \circ f)(1)$ -2

(b) $g(-1)$ -2

9.2.15: Use the functions f and g to find the indicated values.
$f = \{(0, 1), (1, 2), (2, 5), (3, 10), (4, 17))\}$
$g = \{(5, 4), (10, 1), (2, 3), (17, 0), (1, 2)\}$

(a) $f(2)$ 5

(c) $(g \circ f)(1)$ 3

(b) $g(10)$ 1

Find the compositions. $(a) f \circ g$ and $(b) g \circ f$. Then find the domain of each composition.

9.2.19: $f(x) = 3x + 4, g(x) = x - 7$

(a) $(f\circ g)(x) = 3x - 17$ Domain: $(-\infty, \infty)$

(b) $(g\circ f)(x) = 3x - 3$ Domain $(-\infty, \infty)$

9.2.21: $f(x) = \sqrt{x+2}, g(x) = x - 4$

(a) $(f \circ g)(x) = \sqrt{x-2}$, Domain $= [2, \infty)$.

(b) $(g \circ f)(x) = \sqrt{x+2} - 4$, Domain: $[-2, \infty)$.

9.2.23: $f(x) = x^2 + 3, g(x) = \sqrt{x-1}$

a) $f \circ g(x) = \cancel{\quad} x+2$, domain $[1, \infty)$

b) $g \circ f(x) = \sqrt{x^2+2}$ domain $(-\infty, \infty)$

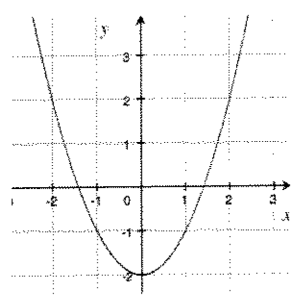
9.2.26: $f(x) = \dfrac{x}{x-4}, g(x) = \sqrt{x}$

a) $(f \circ g)(x) = \dfrac{\sqrt{x}}{\sqrt{x} - 4}$, Domain $[0, 16) \cup (16, \infty)$

b) $(g \circ f)(x) = \sqrt{\dfrac{x}{x-4}}$, Domain $(-\infty, 0] \cup (4, \infty)$

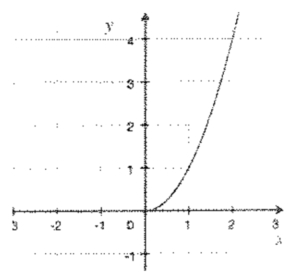
Use the Horizontal Line test to determine if the function is one-to-one and so has an inverse function.

9.2.35: $f(x) = x^2 - 2$



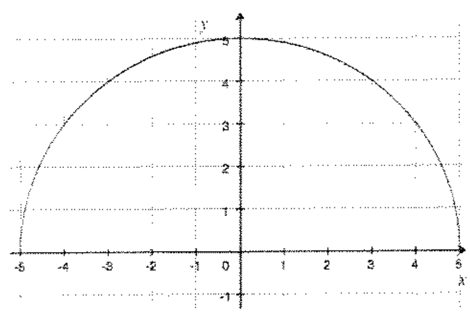
NO.

9.2.37: $f(x) = x^2, x \geq 0$



Yes.

9.2.39: $g(x) = \sqrt{25 - x^2}$



No.

Verify algebraically that the functions f and g are inverse functions of each other.

9.2.41: $f(x) = -6x, g(x) = -\dfrac{1}{6}x$

$f(g(x)) = f(-\frac{1}{6}x) = -6(-\frac{1}{6}x) = x;$

$g(f(x)) = g(-6x) = -\frac{1}{6}(-6x) = x;$

9.2.43: $f(x) = 1 - 2x, g(x) = \dfrac{1}{2}(1 - x)$

$f(g(x)) = f[\frac{1}{2}(1-x)] = 1 - 2[\frac{1}{2}(1-x)] = 1-(1-x) = x;$

$g(f(x)) = g(1-2x) = \frac{1}{2}(1-(1-2x)) = \frac{1}{2}(2x) = x;$

9.2.45: $f(x) = \sqrt[3]{x+1}, g(x) = x^3 - 1$

$f(g(x)) = f(x^3-1) = \sqrt[3]{(x^3-1)+1} = \sqrt[3]{x^3} = x$

$g(f(x)) = g(\sqrt[3]{x+1}) = (\sqrt[3]{x+1})^3 - 1 = x$

Find the inverse function of f. Verify that $f(f^{-1}(x))$ and $f^{-1}(f(x))$ are equal to the identity function.

9.2.49: $f(x) = 5x$

$f^{-1}(x) = \frac{x}{5}$

9.2.55: $f(x) = 5 - x$

$f^{-1}(x) = 5 - x$

9.2.52: $f(x) = \frac{1}{3}x$

$f^{-1}(x) = 3x$

9.2.59: $f(x) = \sqrt[3]{x}$

$f^{-1}(x) = x^3$

Find the inverse function(if it exists).

9.2.64: $g(t) = 6t + 1$

$g^{-1}(t) = \frac{t-1}{6}$

$y = 6t + 1$

$t = 6y + 1$

$t - 1 = 6y$

$\frac{t-1}{6} = y$

9.2.67: $g(x) = x^2 + 4$

Not one to one, inverse doesn't exist.

9.2.70: $h(x) = \sqrt{x+5}$

$h^{-1}(x) = x^2 - 5, \ x \geq 0$

9.2.105: *Ripples* You are standing on a bridge over a calm pond and drop a pebble, causing ripples of concentric circles in the water. The radius (in feet) of the outermost ripple is given by $r(t) = 0.6t$, where t is time in seconds after the pebble hits the water. The area of the circle is given by the function $A(r) = \pi r^2$. Find an equation for the composition $A(r(t))$. What are the input and output of this composite function? What is the area of the circle after 3 seconds.

$A(r(t)) = A(0.6t) = \pi(0.6t)^2 = 0.36\pi t^2$

$A(r(3)) = \pi(3.24) \approx 10.2$ square feet

9.3 Exponential and Logarithmic Functions

Write the logarithmic equation in exponential form.

9.3.1: $\log_7 49$

2

9.3.2: $\log_{11} 121$

2

9.3.3: $\log_2 \frac{1}{32}$

-5

9.3.6: $\log_{10} 10,000$

4

9.3.8: $\log_{64} 4$

$\frac{1}{3}$

9.3.10: $\log_{16} 8$

$\frac{3}{4}$

Write the exponential equation in logarithmic form.

9.3.13: $6^2 = 36$

$\log_6 36 = 2$

9.3.15: $5^{-3} = \frac{1}{125}$

$\log_5 \frac{1}{125} = -3$

9.3.19: $25^{-1/2} = \frac{1}{5}$

$\log_{25} \frac{1}{5} = -\frac{1}{2}$

9.3.22: $6^1 = 6$

$\log_6 6 = 1$

Evaluate the logarithmic without using a calculator. (If not possible, state the reason)

9.3.25: $\log_2 8$

3

9.3.29: $\log_2 \frac{1}{16}$

-4

9.3.32: $\log_6 \frac{1}{216}$

-3

9.3.35: $\log_2(-3)$

Not possible.

9.3.37: $\log_4 1$

0

9.3.38: $\log_3 1$

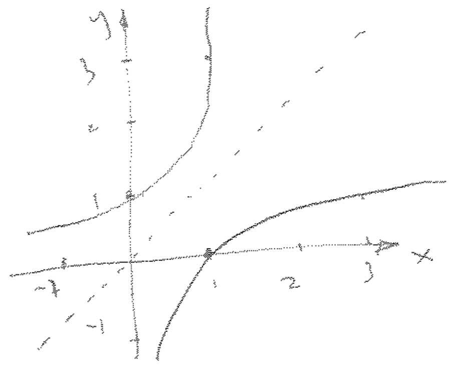
0

9.3.41: $\log_9 3$

$\frac{1}{2}$

Sketch the graph of f, Then use the graph of f to sketch the graph of g.

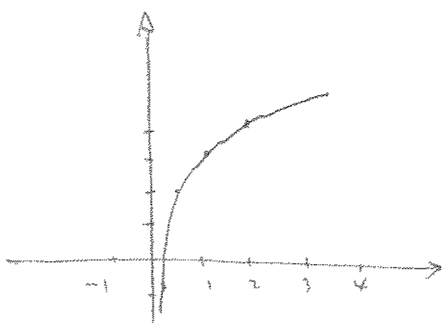
9.3.57: $f(x) = 3^x, g(x) = \log_3 x$



Identify the transformation of the graph of $f(x) = \log_2 x$. Then sketch the graph of h.

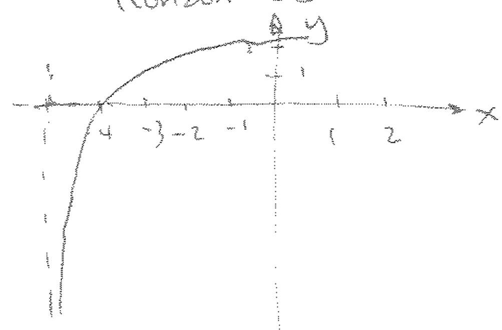
9.3.61: $h(x) = 3 + \log_2 x$

vertical shift;



9.3.64: $h(x) = \log_2(x + 5)$

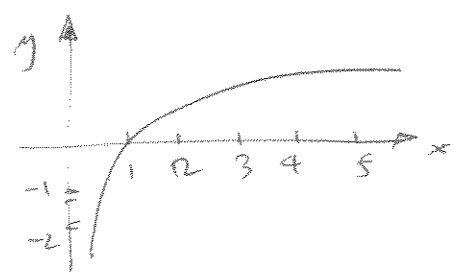
Horizontal shift



Sketch the graph of the function. Identify the vertical asymptote.

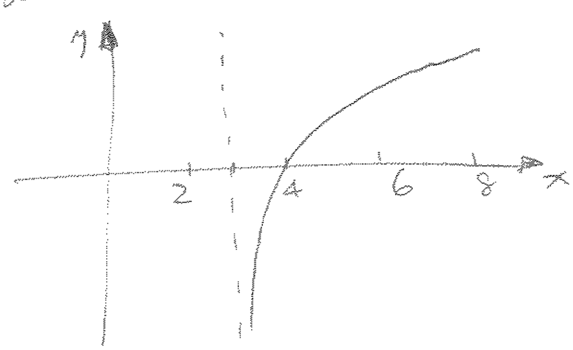
9.3.67: $f(x) = \log_5 x$

Vertical asymptote, $x=0$;

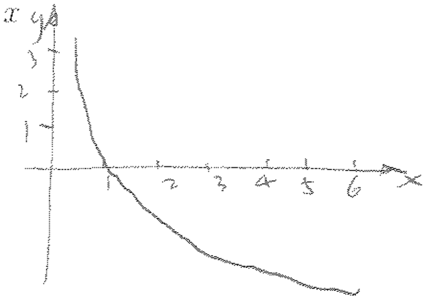


9.3.73: $g(x) = \log_2(x - 3)$

Vertical asymptote, $x=3$

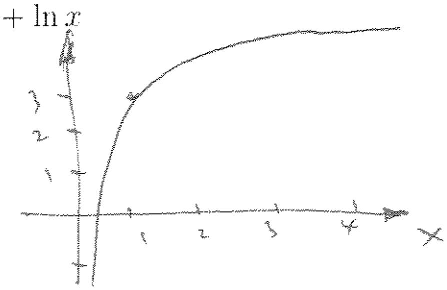


9.3.99: $f(x) = -\ln x$



Vertical asymptote: $x=0$

9.3.103: $f(x) = 3 + \ln x$



Vertical asymptote: $x=0$

Solving Problems:

9.3.125: *American Elk* The antler spread a (in inches) and shoulder height h (in inches) of an adult male American Elk are related by the model

$$h = 116 \log_{10}(a + 40) - 176.$$

Approximate to one decimal place the shoulder height of a male American Elk with an antler spread of 55 inches.

$h = 116 \log_{10}(55+40) - 176$

≈ 53.4 inches

9.3.127: *Compound Interest* The time t in years for an investment to double in value when compounded continuously at interest rate r is given by $t = \frac{\ln 2}{r}$. Complete the table, which shows the "doubling times" for several interest rates.

r	0.07	0.08	0.09	0.10	0.11	0.12
t	9.9	8.7	7.7	6.9	6.3	5.8