ASSIGNMENT 12

DYLAN ZWICK'S MATH 1010 CLASS

7.6 Complex Numbers

Write the number in i-form.

7.6.1: $\sqrt{-4}$ **7.6.5:** $\sqrt{-\frac{4}{25}}$ **7.6.9:** $\sqrt{-8}$ **7.6.15:** $\sqrt{-\frac{18}{25}}$

$2i$ $-\frac{2}{5}i$ $2\sqrt{2}i$ $\frac{3\sqrt{2}}{5}i$

7.6.4: $\sqrt{-49}$ **7.6.6:** $\sqrt{-\frac{9}{64}}$ **7.6.10:** $\sqrt{-75}$

$7i$ $-\frac{3}{8}i$ $5\sqrt{3}i$

Perform the operation(s) and write the result in standard form.

7.6.19: $\sqrt{-16}+\sqrt{-36}$

$10i$

7.6.27: $\sqrt{-12}\sqrt{-2}$

$-2\sqrt{6}$

7.6.20: $\sqrt{-25}-\sqrt{-9}$

$2i$

7.6.30: $\sqrt{-7}\sqrt{-7}$

-7

7.6.21: $\sqrt{-9}-\sqrt{-1}$

$2i$

7.6.35: $\sqrt{-5}(\sqrt{-16}-\sqrt{-10})$

$-4\sqrt{5}+5\sqrt{2}$

7.6.25: $\sqrt{-48}+\sqrt{-12}-\sqrt{-27}$

$3\sqrt{3}i$

7.6.38: $\sqrt{-9}(1+\sqrt{-16})$

$-12+3i$

7.6.42: $(\sqrt{-5})^3$

$-5\sqrt{5}i$

Determine whether the complex numbers are equal.

7.6.43: $\sqrt{1} + \sqrt{-25}$ and $1 + 5i$

Equal

Determine the values of a and b that satisfy the equation.

7.6.47: $3 - 4i = a + bi$

$a=3 \quad b=-4$

7.6.50: $-10 + 12i = 2a + (5b - 3)i$

$a=-5 \quad b=3$

7.6.51: $-4 - \sqrt{-8} = a + bi$

$a=-4 \quad b=-2\sqrt{2}$

Perform the operation(s) and write the result in standard form.

7.6.55: $(4 - 3i) + (6 + 7i)$

$10+4i$

7.6.64: $22 + (-5 + 8i) + 10i$

$17+18i$

7.6.57: $(-4 - 7i) + (-10 - 33i)$

$-14-40i$

7.6.65: $(\frac{4}{3} + \frac{1}{3}i) + (\frac{5}{6} + \frac{7}{6}i)$

$\frac{13}{6} + \frac{3}{2}i$

7.6.60: $17i + (9 - 14i)$

$9+3i$

7.6.69: $15i - (3 - 25i) + \sqrt{-81}$

$-3+49i$

7.6.70: $(-1+i)-\sqrt{2}-\sqrt{-2}$

$(-1-\sqrt{2})+(1-\sqrt{2})i$

Perform the operation and write the result in standard form.

7.6.71: $(3i)(12i)$

-36

7.6.74: $(-2i)(-10i)$

-20

7.6.78: $(8i)^2$

-64

7.6.82: $10(8-6i)$

$80-60i$

7.6.85: $(9-2i)(\sqrt{-4})$

$4+18i$

7.6.89: $(-7+7i)(4-2i)$

$-14+42i$

7.6.92: $(-3-\sqrt{-12})(4-\sqrt{-12})$

$-24-2\sqrt{3}i$

7.6.94: $(7+i)^2$

$48+14i$

Simplify the expression.

7.6.99: i^7-i

7.6.102: $i^{35}-i$

Multiply the number by its complex conjugate and simplify.

7.6.109: $2+i$

5

7.6.115: $10i$

100

Write the quotient in standard form.

7.6.123: $\frac{20}{2i} - 10i$

7.6.135: $\frac{5-i}{5+i}$ $\frac{12}{13} - \frac{5}{13}i$

7.6.130: $\frac{6i+3}{3i}$ $2-i$

Determine whether each number is a solution of the equation.

7.6.145: $x^2 + 2x + 5 = 0$ (a) $x = -1 + 2i$ (b) $x = -1 - 2i$

Solution Solution

7.6.147: $x^3 + 4x^2 + 9x + 36 = 0$ (a) $x = -4$ (b) $x = -3i$

Solution Solution

7.6.158: *True or False"* Some numbers are both real and imaginary. Justify your answer.

False, a number is either real or imaginary.
For example, 2 is real, $2i$ is imaginary, and $2+2i$ is imaginary

8.1 Solving Quadratic Equations: Factoring and Special Forms

Solve the equation by factoring.

8.1.1: $x^2 - 15x + 54 = 0$

$x = 6, 9$

8.1.3: $x^2 - x - 30 = 0$

$(x-6)(x+5)=0$

$x=6,-5$

8.1.5: $x^2 + 4x = 45$

$x=-9, 5$

8.1.8: $x^2 + 60x + 900 = 0$

$x=-30$

8.1.10: $8x^2 - 10x + 3 = 0$

$x=\frac{1}{2}, \frac{3}{4}$

8.1.14: $16x(x-8) - 12(x-8) = 0$

$x=\frac{3}{4}, 8$

8.1.18: $(5+u)(2+u) = 4$

$10+5u+2u+u^2=4$

$u^2+7u+6=0$

$(u+6)(u+1)=0$

$u=-6,-1$

Solve the equation by using the Square Root Property.

8.1.21: $x^2 = 49$

± 7

8.1.23: $6x^2 = 54$

± 3

8.1.27: $\frac{w^2}{4} = 49$

± 14

8.1.38: $(y+4)^2 = 27$

$y + 4 = \pm\sqrt{27}$

$y = -4 \pm 3\sqrt{3}$

Solve the equation by using the Square Root Property.

8.1.43: $z^2 = -36$

$\pm 6i$

8.1.45: $x^2 + 4 = 0$

$\pm 2i$

8.1.50: $(x+5)^2 = -81$

$-5 \pm 9i$

Find all real and complex solutions of the quadratic equation.

8.1.65: $2x^2 - 5x = 0$

$0, \frac{5}{2}$

8.1.69: $x^2 - 900 = 0$

$x^2 = 900$

$x = \pm 30$

8.1.70: $z^2 - 256 = 0$

±16

8.1.72: $z^2 + 256 = 0$

±16i

8.1.76: $(y + 12)^2 - 400 = 0$

−32, 8

Solve the equation of quadratic form.

8.1.101: $x^4 - 5x^2 + 4 = 0$

$(x^2-4)(x^2-1)=0$

$x = \pm 2, \pm 1$

8.1.105: $(x^2 - 4)^2 + 2(x^2 - 4) - 3 = 0$

$[(x^2-4)+3][(x^2-4)-1]=0$

$(x^2-1)(x^2-5)=0$

$x = \pm 1, \pm\sqrt{5}$

8.1.107: $x - 3\sqrt{x} - 4 = 0$

$(\sqrt{x})^2 - 3\sqrt{x} - 4 = 0$

$(\sqrt{x}-4)(\sqrt{x}+1)=0$

$\sqrt{x} = 4$, $x = 16$; $\sqrt{x}+1=0$ No solution

8.1.114: $5x^{2/3} - 13x^{1/3} + 6 = 0$

$$(5x^{\frac{1}{3}} - 3)(x^{\frac{1}{3}} - 2) = 0$$

$$x = \frac{27}{125},\ 8$$

8.1.120: $x^{1/3} + 2x^{1/6} - 3 = 0$

$$(x^{\frac{1}{6}} + 3)(x^{\frac{1}{6}} - 1) = 0$$

$x^{\frac{1}{6}} + 3 = 0$ No solution.

$x^{\frac{1}{6}} - 1 = 0,\ x = 1$;

8.1.131: *Unisphere* The Unisphere is the world's largest man-made globe. It was built as the symbol of the 1964-1965 New York World's Fair. A sphere with the same diameter as the Unisphere globe would have a surface area of 45,239 square feet. What is the diameter of the Unisphere?

SA (Surface area of a sphere) $= 4\pi r^2$

$$45{,}239 = 4\pi\left(\frac{d}{2}\right)^2 = \pi d^2$$

$$\Rightarrow d = \sqrt{\frac{45{,}239}{\pi}} \qquad d \approx 120 \text{ feet}$$

8.2 Completing the Square

Add a term to the expression so that it becomes a perfect square trinomial.

8.2.1: $x^2 + 8x +$

16

8.2.10: $y^2 - 11y +$

$\frac{121}{4}$

8.2.3: $y^2 - 20y +$

100

Solve the equation first by completing the square and then by factoring.

8.2.17: $x^2 - 20x = 0$

$(x-10)^2 = 100$

$x(x-20)=0$

$x = 20, 0$

8.2.25: $x^2 + 7x + 12 = 0$

$x^2+7x+\frac{49}{4} = -12+\frac{49}{4}$

$(x+\frac{7}{2})^2 = \frac{1}{4}$

$x=-3,-4$

$x^2+7x+12=0$

$(x+4)(x+3)=0$ $x=-3,-4$

8.2.19: $x^2 + 6x = 0$

$x^2+6x+9=0+9$

$(x+3)^2 = 9$

$x=-6, 0$

$x^2+6x=0$

$x(x+6)=0$

$x=0,-6$

8.2.30: $3x^2 - 3x - 6 = 0$

$x^2 - x - 2 = 0$

$(x-\frac{1}{2})^2 = \frac{9}{4}$

$(x-2)(x+1)=0$ $x=-1,2$

Solve the equation by completing the square. Give the solutions in exact form and in decimal form rounded to two decimal places. (Solutions may be complex numbers)

8.2.33: $x^2 - 4x - 3 = 0$

$(x-2)^2 = 7$

$x = 2 \pm \sqrt{7}$

$x \approx 4.65, -0.65$

8.2.54: $y^2 + 5y + 9 = 0$

$\left(y + \frac{5}{2}\right)^2 = -\frac{11}{4}$

$y = -\frac{5}{2} \pm \frac{\sqrt{11}}{2} i$

$y \approx -2.50 + 1.66i$
$-2.50 - 1.66i$

8.2.40: $x^2 - 4x = -9$

$x^2 - 4x + 4 = -9 + 4$

$(x-2)^2 = -5$

$x - 2 = \pm\sqrt{-5}$

$x = 2 \pm \sqrt{5}i$

$x \approx 2 + 2.24i, 2 - 2.24i$

8.2.59: $2x^2 + 8x + 3 = 0$

$x^2 + 4x + 4 = -\frac{3}{2} + 4$

$(x+2)^2 = \frac{5}{2}$

$x \approx -0.42, -3.58$

8.2.49: $z^2 + 4z + 13 = 0$

$(z+2)^2 = -9$

$z = -2 \pm 3i$

8.2.60: $3x^2 - 24x - 5 = 0$

$x^2 - 8x - \frac{5}{3} = 0$

$(x-4)^2 = \frac{53}{3}$

$x = 4 \pm \frac{\sqrt{159}}{3}$

$x \approx 8.20, -0.20$

8.2.53: $a^2 + 7a + 11 = 0$

$\left(a + \frac{7}{2}\right)^2 = \frac{5}{4}$

$a = -\frac{7}{2} \pm \frac{\sqrt{5}}{2}$

$a \approx -2.38, -4.62$

8.2.61: $3x^2 + 9x + 5 = 0$

$x^2 + 3x + \frac{9}{4} = -\frac{5}{3} + \frac{9}{4}$

$\left(x + \frac{3}{2}\right)^2 = \frac{7}{12}$

$x = -\frac{3}{2} \pm \frac{\sqrt{21}}{6}$

$x \approx -0.74, -2.26$

8.2.66: $7u^2 - 8u - 3 = 0$

$(u - \frac{4}{7})^2 = \frac{37}{49}$

$u = \frac{4 \pm \sqrt{37}}{4}$

$u \approx 1.44, -0.30$

8.2.69: $0.1x^2 + 0.5x = -0.2$

$(x + \frac{5}{2})^2 = \frac{17}{4}$

$x = -\frac{5}{2} \pm \frac{\sqrt{17}}{2}$

$x \approx 0.44, -4.56$

Find the real solutions.

8.2.73: $\frac{x}{2} - \frac{1}{x} = 1$

$2x(\frac{x}{2} - \frac{1}{x}) = 2x$

$x^2 - 2x - 2 = 0$

$x = 1 \pm \sqrt{3}$

8.2.94: *Revenue* The revenue R (in dollars) from selling x golf clubs is given by

$$R = x(150 - \frac{1}{10}x).$$

Find the number of golf clubs that must be sold to produce a revenue of \$ 15,033.60.

$-\frac{1}{10}x^2 + 150x - R = 0$

$R = 15,033.60$

$x^2 - 1500x + 150,336 = 0$

$x = 108, 1392$