ASSIGNMENT 10

DYLAN ZWICK'S MATH 1010 CLASS

6.6 Solving Rational Equations

Determine whether each value of x is a solution of the equation.

6.6.1: $\frac{x}{3} - \frac{x}{5} = \frac{4}{3}$

(a) $x = 0$ (b) $x = -2$ (c) $x = \frac{1}{8}$ (d) $x = 10$

Not a Solution Not a Solution Not a Solution Solution

Solve the equation.

6.6.5: $\frac{x}{6} - 1 = \frac{2}{3}$

$6\left(\frac{x}{6} - 1\right) = \left(\frac{2}{3}\right) \cdot 6$

$x - 6 = 4$

$x = 10$

6.6.7: $\frac{1}{4} = \frac{z+1}{8}$

$4(z+1) = 8 \cdot 1$

$4z + 4 = 8$

$4z = 4$

$z = 1$

6.6.11: $\frac{z+2}{3} = 4 - \frac{z}{12}$

$12\left(\frac{z+2}{3}\right) = \left(4 - \frac{z}{12}\right) \cdot 12$

$4(z+2) = 48 - z$

$5z = 40$

$z = 8$

6.6.12: $\frac{2y-9}{6} = 3y - \frac{3}{4}$

$y = -\frac{9}{32}$

6.6.14: $\frac{4x-2}{7} - \frac{5}{14} = 2x$

$x = -\frac{9}{20}$

6.6.15: $\frac{t}{2} = 12 - \frac{3t^2}{2}$

$2\left(\frac{t}{2}\right) = \left(12 - \frac{3t^2}{2}\right) \cdot 2$

$t = 24 - 3t^2$

$3t^2 + t - 24 = 0$

$(3t-8)(t+3) = 0$

$t = -3, \frac{8}{3}$

6.6.18: $\frac{z-4}{9} - \frac{3z+1}{18} = \frac{3}{2}$

$z = -36$

6.6.20: $\frac{u-2}{6} + \frac{2u+5}{15} = 3$

$u = 10$

6.6.22: $\frac{2x-7}{10} - \frac{3x+1}{5} = \frac{6-x}{5}$

$x = -\frac{21}{2}$

6.6.23: $\dfrac{9}{25-y} = -\dfrac{1}{4}$

$y = 61$

6.6.25: $5 - \dfrac{12}{a} = \dfrac{5}{3}$

$3a\left(5 - \frac{12}{a}\right) = \frac{5}{3} \cdot 3a$

$a = \frac{18}{5}$

$15a - 36 = 5a$

$10a = 36$

6.6.28: $\dfrac{5}{3} = \dfrac{6}{7x} + \dfrac{2}{x}$

$x = \frac{12}{7}$

6.6.30: $\dfrac{7}{8} - \dfrac{16}{t-2} = \dfrac{3}{4}$

$8(t-2)\left(\frac{7}{8} - \frac{16}{t-2}\right) = \frac{3}{4} \cdot 8(t-2)$

$t = 130$

$7(t-2) - 16 \cdot 8 = 6(t-2)$

$7t - 142 = 6t - 12$

$t = 130$

6.6.32: $\dfrac{10}{x+4} = \dfrac{15}{4(x+1)}$

$x = \frac{4}{5}$

6.6.35: $\dfrac{3}{x+2} - \dfrac{1}{5} = \dfrac{1}{5x}$

$x = \dfrac{4}{3}$

6.6.39: $\dfrac{t}{4} = \dfrac{4}{t}$

$t = \pm 4$

6.6.45: $\dfrac{4}{x(x-1)} + \dfrac{3}{x} = \dfrac{4}{x-1}$

No solution

6.6.47: $\dfrac{2x}{5} = \dfrac{x^2 - 5x}{5x}$

$5x\left(\dfrac{2x}{5}\right) = \left(\dfrac{x^2-5x}{5x}\right) \cdot 5x$

$2x^2 = x^2 - 5x$

$x^2 + 5x = 0$

$x = 0$ or $x = -5$

$x = 0$ is not a solution.

$x = -5$

6.6.49: $\dfrac{y+1}{y+10} = \dfrac{y-2}{y+4}$

$y = 8$

6.6.50: $\dfrac{x-3}{x+1} = \dfrac{x-6}{x+5}$

$x = \dfrac{9}{7}$

6.6.55: $\dfrac{4}{2x+3} + \dfrac{17}{5x-3} = 3$

$x = -\dfrac{11}{10}, 2$

6.6.57: $\dfrac{2}{x-10} - \dfrac{3}{x-2} = \dfrac{6}{x^2-12x+20}$

$x = 20$

6.6.61: $\dfrac{x}{x-2} + \dfrac{3x}{x-4} = -\dfrac{2(x-6)}{x^2-6x+8}$

$(x-2)(x-4)\left(\dfrac{x}{x-2} + \dfrac{3x}{x-4}\right) = \dfrac{-2(x-6)}{x^2-6x+8}(x-2)(x-4)$

$x(x-4) + (x-2)\cdot 3x = -2(x-6)$

$x^2 - 4x + 3x^2 - 6x = -2x + 12$

$4x^2 - 8x - 12 = 0$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

6.6.65: $\frac{x}{3} = \frac{1+\frac{4}{x}}{1+\frac{2}{x}}$ $\Rightarrow$ $x^2 - x - 12 = 0$ $x = -3, 4$

6.6.83 ***Painting***: A painter can paint a fence in 4 hours, while his partner can paint the fence in 6 hours. How long would it take to paint the fence if both worked together?

2 hours 24 minutes

6.7 Applications and Variations

Write a model for the statement.

6.7.1: l varies directly as V.

$l = kV$

6.7.2: C varies directly as r.

$C = kr$

6.7.3: V directly proportional to t

$V = kt$

6.7.4: A is directly proportional to w.

$A = kw$

6.7.5: u is directly proportional to the square of v.

$u = kv^2$

6.7.6: s varies directly as the cube of t.

$S = kt^3$

6.7.7: p varies inversely as d.

$p = \frac{k}{d}$

6.7.8: S varies inversely as the square of v.

$S = k/v^2$

6.7.10: P is inversely proportional to the square root of $1 + r$

$P = \frac{k}{\sqrt{1+r}}$

6.7.13 ***Boyle's Law*:** If the temperature of a gas is not allowed to change, its absolute pressure P is inversely proportional to its volume$V$.

$P = k/V$

6.7.14 ***Newton's Law of Universal Gravitation*:** The gravitational attraction F between two particles of masses m_1 and m_2 is directly proportional to the product of the masses and inversely proportional to the square of the distance r between the particles.

$F = k\frac{m_1 m_2}{r^2}$

Write a verbal sentence using variation terminology to describe the formula.

6.7.18 ***Volume of a Sphere*:** $V = \frac{4}{3}\pi r^3$

Volume varies directly as the cube of the radius

Find the constant of proportionality and write an equation that relates the variables.

6.7.21: s varies directly as t, and $s = 20$ when $t = 4$.

$s = 5t$

6.7.25: n varies inversely as m, and $n = 32$ when $m = 1.5$

$n = 48/m$

6.7.32: z is directly proportional to x and inversely proportional to the square root of y, and $z = 720$ when $x = 48$ and $y = 81$.

$$z = \frac{135x}{\sqrt{y}}$$

6.7.43: *Average Speeds* You and a friend jog for the same amount of time. You jog 10 miles and your friend jogs 12 miles. Your friends's average speed is 1.5 miles per hour faster than yours. What are the average speeds of you and your friend?

your rate $= r$ $\qquad \frac{10}{r} = \frac{12}{r+1.5} \Rightarrow r = 7.5$

your average speed : 7.5 miles per hour.

your friend's average speed : 9 miles per hour

6.7.47: *Working Rate* It takes a lawn care company 60 minutes to complete a job using only a riding mower, or 45 minutes using the riding mower and a push mower. How long does the job take using only the push mower.

Riding mower rate $= \frac{1}{60}$

Push mower rate $= \frac{1}{x}$

Rate together $= \frac{1}{45}$

$\frac{1}{60} + \frac{1}{x} = \frac{1}{45} \Rightarrow x = 180$

So it takes 180 minutes using only push mower.

6.7.52: *Learning Curve* A psychologist observes that the number of lines N of a poem that a four-year-old child can memorize depends on the number x of short sessions spent on the task, according to the mode $N = \dfrac{20x}{x+1}$.

(a) What is the domain of the function?

$\{1, 2, 3, 4 \ldots\}$

(b) Use a graphing calculator to graph the function.

(c) Use the graph to determine the number of sessions needed for a child to memorize 15 lines of poem.

3 sessions will be needed

(d) Verity the result of part(c) algebraically.

$15 = \frac{20X}{X+1} \Rightarrow X = 3$

6.7.69: *Revenue* The weekly demand for a company's frozen pizzas varies directly as the amount spent on advertising and inversely as the price per pizza. At \$5 per pizza, when \$500 is spent each week on ads, the demand is 2000 pizzas. If advertising is increased to \$600, what price will yield a demand of 2000 pizzas? Is this increase worthwhile in terms of revenue?

$D = K \frac{a}{p}$

$2000 = K \frac{500}{5} \Rightarrow K = 20$ so $D = \frac{20a}{p}$

$2000 = \frac{20 \cdot 600}{p} \Rightarrow p = 6$

the price is \$6.

7.1 Radicals and Rational Exponents

Find the root if it exists.

7.1.1: $\sqrt{64}$ 8

7.1.4: $\sqrt{-25}$ Not a Real number

7.1.7: $\sqrt{-1}$ Not a real number

7.1.2: $-\sqrt{100}$ -10

7.1.5: $\sqrt[3]{-27}$ -3

7.1.8: $-\sqrt{-1}$ Not a real number

State whether the number is a perfect square, a perfect cube,or neither.

7.1.9: 49 — Perfect square

7.1.13: 96 — Neither

7.1.14: 225 — Perfect square

Find all of the square roots of the perfect square.

7.1.15: 25 — ± 5

7.1.16: 121 — ± 11

Evaluate the radical expression without using a calculator. If not possible, state the reason.

7.1.39: $\sqrt{8^2}$ — 8

7.1.62: $(\sqrt[3]{-6})^3$ — -6

7.1.42: $\sqrt{(-12)^2}$ — 12

Evaluate without using a calculator.

7.1.75: $-36^{1/2}$ — -6

7.1.78: $81^{-3/4}$ — $\frac{1}{27}$

7.1.86: $(8^2)^{3/2}$ — 512

Rewrite the expression using rational exponents.

7.1.91: $x\sqrt[3]{x^6}$ — x^3

7.1.99: $\sqrt[3]{x^2} \cdot \sqrt[3]{x^7}$ — x^3

7.1.95: $\frac{\sqrt{x}}{\sqrt{x^3}}$ — $x^{-1} = \frac{1}{x}$

7.1.105: $z^2\sqrt{y^5 z^4}$ — $y^{\frac{5}{2}} z^4$

Simplify the expression.

7.1.107: $3^{1/4} \cdot 3^{3/4}$ — 3

7.1.125: $\frac{(x+y)^{3/4}}{\sqrt[4]{x+y}}$ — $(x+y)^{\frac{1}{2}}$