Math 2210 - Exam 2

University of Utah

Fall 2008

Name: _____

1. (10 points) *Partial Derivatives*

   Calculate the partial derivatives, $f_x(x, y)$ and $f_y(x, y)$, of the following functions:

   (a) (3 points)

   $$f(x, y) = e^x \cos(y)$$

(b) (3 points)

$$f(x, y) = y \cos(x^2 + y^2)$$

(c) (4 points)

$$f(x, y) = e^{x^2 - y^2}$$

2. (10 points) *Limits*

   Determine each of the following limits, or state it does not exist and give an explanation as to why:

   (a) (3 points)

$$\lim_{(x,y)\to(-2,1)} (xy^3 - xy + 3y^2)$$

   (b) (3 points)

$$\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2+y^2}}$$

(c) (4 points)

$$\lim_{(x,y)\to(0,0)} \frac{xy^2}{x^2+y^4}$$

3. (15 points) *Gradients and Directional Derivatives*

   For each problem, find the directional derivative of f at the point $\mathbf{p}$ in the direction of $\mathbf{a}$:

   (a) (4 points)

$$f(x, y) = x^2 - 3xy + 2y^2$$
$$\mathbf{p} = (-1, 2), \mathbf{a} = 2\mathbf{i} - \mathbf{j}$$

   (b) (5 points)

$$f(x, y) = y^2 \ln x$$
$$\mathbf{p} = (1, 4), \mathbf{a} = \mathbf{i} - \mathbf{j}$$

(c) (6 points)

Find the unit vector in the direction in which f increases most raplidly at $\mathbf{p}$. What is the rate of change in this direction?

$$f(x, y, z) = x^2yz$$
$$\mathbf{p} = (1, -1, 2)$$

4. (5 points) *The Chain Rule*

   The part of a tree normally sawed into lumber is the trunk, a solid shaped approximately like a right circular cylinder. If the radius of the trunk of a certain tree is growing $\frac{1}{2}$ inch per year and the height is increasing 8 inches per year, how fast is the volume increasing when the radius is 20 inches and the height is 400 inches? Express your answer in board feet per year (1 board foot = 1 inch by 12 inches by 12 inches).

5. (5 points) *Tangent Planes*

   Find the equation of the tangent plane to the given surface at the indicated point.

$$x^2 - y^2 + z^2 + 1 = 0$$
$$\mathbf{p} = (1, 3, \sqrt{7})$$

6. (5 points) *Extrema*

   Find all critical points of the given function, and indicate whether each such point gives a local maximum, a local minimum, or a saddle point.

$$f(x, y) = 2x^4 - x^2 + 3y^2$$