Math 2210 - Assignment 4

Dylan Zwick

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1 Sections 11.8 and 11.9

1.1 Section 11.8

11.8.1 Name and sketch the graph of the following equation in three-space.

$$4x^2 + 36y^2 = 144$$

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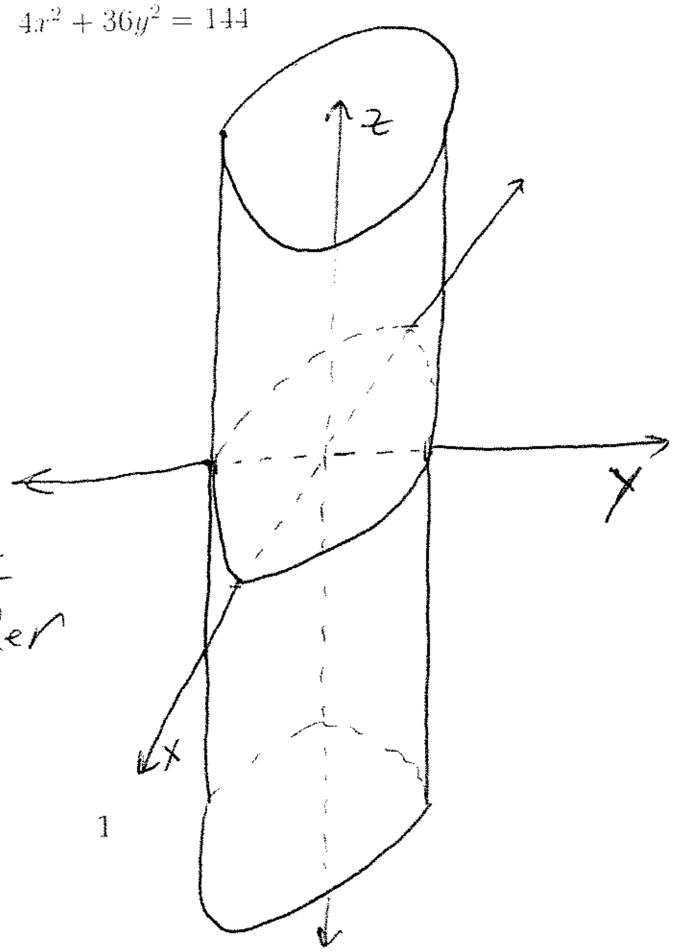
$$\Rightarrow \frac{x^2}{36} + \frac{y^2}{4} = 1$$

$$\Rightarrow \frac{x^2}{6^2} + \frac{y^2}{2^2} = 1$$

Elliptic Cylinder



11.8.9 Name and sketch the graph of the following equation in three-space.

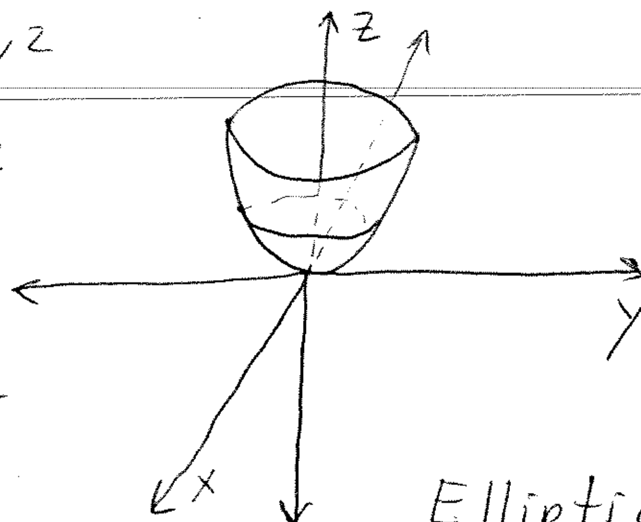
$$4x^2 + 16y^2 - 32z = 0$$

$$4x^2+16y^2-32z=0$$

$$\Rightarrow 32z = 4x^2+16y^2$$

$$\Rightarrow z = \frac{x^2}{8} + \frac{y^2}{2}$$

$$\Rightarrow z = \frac{x^2}{(2\sqrt{2})^2} + \frac{y^2}{(\sqrt{2})^2}$$



Elliptic Paraboloid

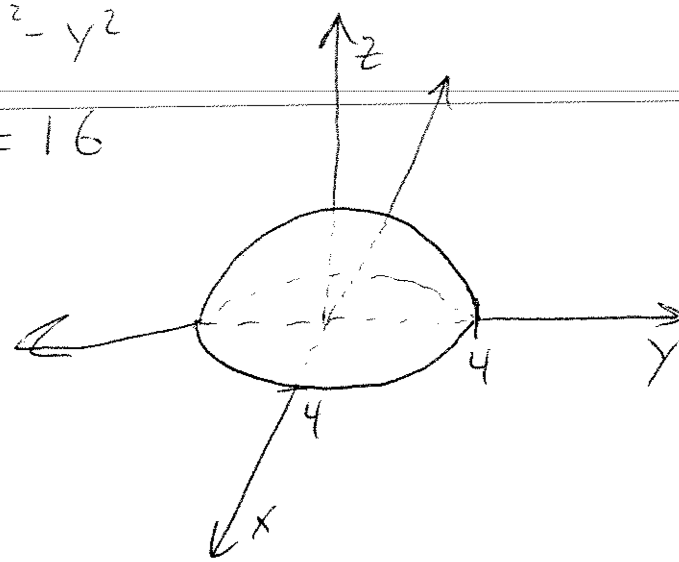
11.8.19 Name and sketch the graph of the following equation in three-space.

$$z = \sqrt{16 - x^2 - y^2}$$

$z = \sqrt{16 - x^2 - y^2}$

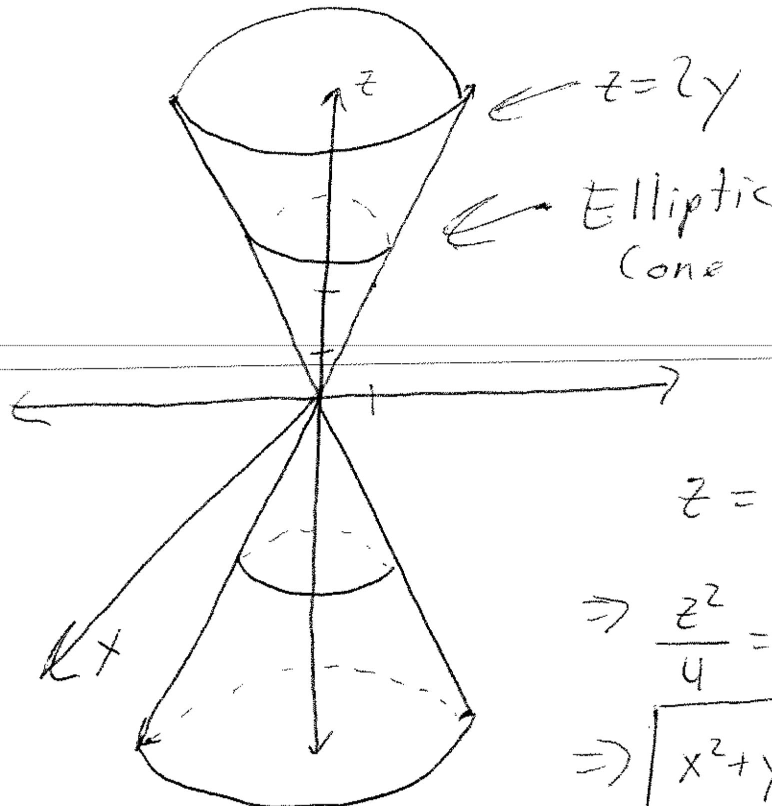
$\Rightarrow z^2 = 16 - x^2 - y^2$

$\Rightarrow x^2 + y^2 + z^2 = 16$



Upper hemisphere of a sphere of radius 4.

11.8.28 Find the equation of the surface that results when the curve $z = 2y$ in the yz-plane is revolved about the z-axis.



$$z = 2\sqrt{x^2 + y^2}$$

$$\Rightarrow \frac{z^2}{4} = x^2 + y^2$$

$$\Rightarrow \boxed{x^2 + y^2 - \frac{z^2}{4} = 0}$$

1.2 Section 11.9

11.9.2 Change the following from cylindrical to spherical coordinates.

1. $(1, \pi/2, 1)$

$\rho = \sqrt{r^2 + z^2} = \sqrt{1^2 + 1^2} = \sqrt{2}$

$\theta = \theta = \pi/2$

$\phi = \tan^{-1}\left(\frac{r}{z}\right) = \tan^{-1}\left(\frac{1}{1}\right) = \pi/4$

$$\boxed{(\sqrt{2}, \pi/2, \pi/4)}$$

2. $(-2, \pi/4, 2)$

$\rho = \sqrt{(-2)^2 + 2^2} = 2\sqrt{2}$

$\theta = \pi/4$

$\phi = \tan^{-1}\left(\frac{-2}{2}\right) = -\pi/4$

Note: It would be better to write this point as:

$$\boxed{(2\sqrt{2}, 5\pi/4, \pi/4)}$$

11.9.5 Change the following from Cartesian to spherical coordinates.

1. $(2, -2\sqrt{3}, 4)$

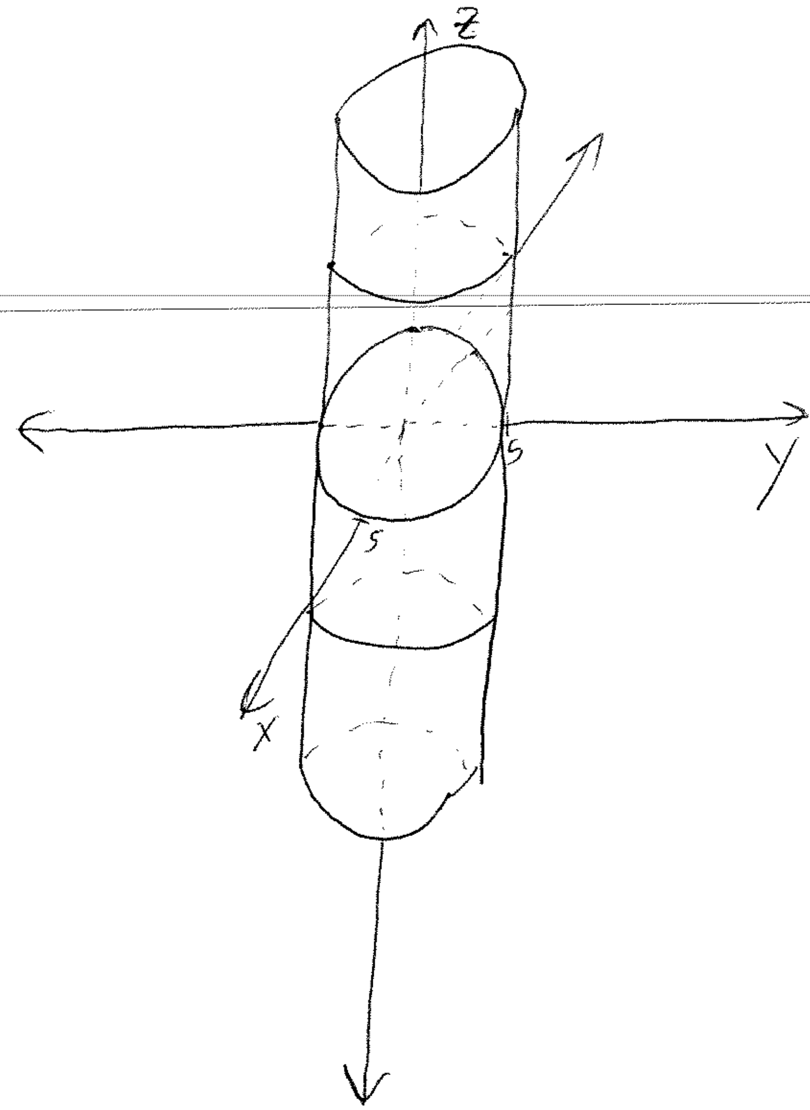
$$\rho = \sqrt{2^2 + (-2\sqrt{3})^2 + 4^2}$$
$$= \sqrt{4 + 12 + 16} = 4\sqrt{2}$$
$$\theta = \tan^{-1}\left(\frac{-2\sqrt{3}}{2}\right) = \frac{5\pi}{3}$$
$$\phi = \cos^{-1}\left(\frac{4}{4\sqrt{2}}\right) = \frac{\pi}{4}$$
$$\boxed{\left(4\sqrt{2}, \frac{5\pi}{3}, \frac{\pi}{4}\right)}$$

2. $(-\sqrt{2}, \sqrt{2}, 2\sqrt{3})$.

$$\rho = \sqrt{(-\sqrt{2})^2 + (\sqrt{2})^2 + (2\sqrt{3})^2}$$
$$= \sqrt{2 + 2 + 12} = \sqrt{16} = 4$$
$$\theta = \tan^{-1}\left(\frac{\sqrt{2}}{-\sqrt{2}}\right) = \frac{3\pi}{4}$$
$$\phi = \cos^{-1}\left(\frac{2\sqrt{3}}{4}\right) = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$$
$$\boxed{\left(4, \frac{3\pi}{4}, \frac{\pi}{6}\right)}$$

11.9.7 Sketch the graph of the given cylindrical or spherical coordinates.

$$r = 5$$



11.9.17 Change the given equation into the equivalent equation in cylindrical coordinates:

$$x^2 + y^2 = 9$$

$x^2 + y^2 = r^2$

$\Rightarrow r^2 = 9$

$\Rightarrow \boxed{r = 3}$

11.9.25 Change the given equation into the equivalent equation in cylindrical coordinates.

$$x + y = 4$$

$x = r\cos\theta$

$y = r\sin\theta$

$\Rightarrow \quad r\cos\theta + r\sin\theta = 4$

$$\Rightarrow \quad \boxed{r = \frac{4}{\cos\theta + \sin\theta}}$$

11.9.29 Change the given equation into the equivalent equation in Cartesian coordinates.

$$r^2 \cos 2\theta = z$$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$\Rightarrow r^2(\cos^2\theta - \sin^2\theta) = z$

$\Rightarrow x^2 - y^2 = z$

$\boxed{x^2 - y^2 = z}$