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Math 2210
Assignment #2
Solutions

11.3.1

Let $\vec{a} = -2\hat{i} + 3\hat{j}$, $\vec{b} = 2\hat{i} - 3\hat{j}$, and $\vec{c} = -5\hat{j}$.
Find each of the following:

a) $2\vec{a} - 4\vec{b}$

$$= \boxed{-12\hat{i} + 18\hat{j}}$$

b) $\vec{a} \cdot \vec{b} = (-2(2) + 3(-3)) = \boxed{-13}$

c) $\vec{a} \cdot (\vec{b} + \vec{c})$

$$= (-2\hat{i} + 3\hat{j}) \cdot (2\hat{i} - 3\hat{j} + (-5\hat{j}))$$

$$= (-2\hat{i} + 3\hat{j}) \cdot (2\hat{i} - 8\hat{j}) = -4 - 24 = \boxed{-28}$$

d) $(-2\vec{a} + 3\vec{b}) \cdot 5\vec{c}$

$$= (-2(-2\hat{i} + 3\hat{j}) + 3(2\hat{i} - 3\hat{j})) \cdot (5(-5\hat{j}))$$

$$= (10\hat{i} - 15\hat{j}) \cdot (-25\hat{j}) = \boxed{375}$$

e) $\|\vec{a}\| \vec{c} \cdot \vec{a}$ $\qquad \|\vec{a}\| = \sqrt{(-2)^2 + 3^2} = \sqrt{13}$

$$\vec{c} \cdot \vec{a} = (-5\hat{j}) \cdot (-2\hat{i} + 3\hat{j}) = -15$$

$$\Rightarrow \|\vec{a}\| \vec{c} \cdot \vec{a} = \boxed{-15\sqrt{13}}$$

f) $\vec{b} \cdot \vec{b} = (2\hat{i} - 3\hat{j}) \cdot (2\hat{i} - 3\hat{j}) = 13$

$$\vec{b} \cdot \vec{b} - \|\vec{b}\| = \|\vec{b}\|^2 - \|b\| = 13 - \sqrt{13} = \boxed{\sqrt{13}(\sqrt{13} - 1)}$$

11.3.6.

$\vec{a} = \langle \sqrt{2}, \sqrt{2}, 0 \rangle \quad \vec{b} = \langle 1, -1, 1 \rangle, \quad \vec{c} = \langle -2, 2, 1 \rangle.$

a) $\vec{a} \cdot \vec{c} = \langle \sqrt{2}, \sqrt{2}, 0 \rangle \cdot \langle -2, 2, 1 \rangle$

$= -2\sqrt{2} + 2\sqrt{2} + 0 - \cancel{1} = \boxed{0}$

b) $(\vec{a} - \vec{c}) \cdot \vec{b} = \langle \sqrt{2} + 2, \sqrt{2} - 2, -1 \rangle \cdot \langle 1, -1, 1 \rangle$

$= \sqrt{2} + 2 - (\sqrt{2} - 2) - 1$

$= \boxed{3}$

c) $\vec{a} / \|\vec{a}\| = \dfrac{1}{\sqrt{(\sqrt{2})^2 + (\sqrt{2})^2 + 0^2}} \langle \sqrt{2}, \sqrt{2}, 0 \rangle$

$= \frac{1}{2} \langle \sqrt{2}, \sqrt{2}, 0 \rangle = \boxed{\left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0 \right\rangle}$

d) $(\vec{b} - \vec{c}) \cdot \vec{a} = \langle 3, -3, 0 \rangle \cdot \langle \sqrt{2}, \sqrt{2}, 0 \rangle$

$= 3\sqrt{2} - 3\sqrt{2} + 0 = \boxed{0}$

e) $\dfrac{\vec{b} \cdot \vec{c}}{\|\vec{b}\| \|\vec{c}\|} = \dfrac{\langle 1, -1, 1 \rangle \cdot \langle -2, 2, 1 \rangle}{\sqrt{1^2 + (-1)^2 + 1^2} \sqrt{(-2)^2 + 2^2 + 1^2}} = \dfrac{-3}{\sqrt{3} \sqrt{9}} = \boxed{-\dfrac{1}{\sqrt{3}}}$

f) $\vec{a} \cdot \vec{a} - \|\vec{a}\|^2$

$\vec{a} \cdot \vec{a} = \|\vec{a}\|^2$ by definition $\Rightarrow \vec{a} \cdot \vec{a} - \|\vec{a}\|^2 = \boxed{0}$

11.3.7.

For vectors $\vec{a}$, $\vec{b}$, and $\vec{c}$ find the angle between each pair of vectors.

Between $\vec{a}$ and $\vec{b}$:

$$\vec{a} \cdot \vec{b} = \langle \sqrt{2}, \sqrt{2}, 0 \rangle \cdot \langle 1, -1, 1 \rangle = \sqrt{2} - \sqrt{2} + 0 = 0.$$

$$\|\vec{a}\| = \sqrt{(\sqrt{2})^2 + (\sqrt{2})^2 + 0^2} = \sqrt{4} = 2$$

$$\|\vec{b}\| = \sqrt{1^2 + (-1)^2 + 1^2} = \sqrt{3}$$

$$\theta = \cos^{-1}\left(\frac{0}{2\sqrt{3}}\right) = \cos^{-1}(0) = \boxed{90^\circ}$$

Between $\vec{a}$ and $\vec{c}$:

$$\vec{a} \cdot \vec{c} = \langle \sqrt{2}, \sqrt{2}, 0 \rangle \cdot \langle -2, 2, 1 \rangle = -2\sqrt{2} + 2\sqrt{2} + 0 = 0$$

$$\|\vec{c}\| = \sqrt{(-2)^2 + 2^2 + 1^2} = \sqrt{9} = 3$$

$$\theta = \cos^{-1}\left(\frac{0}{(2)(3)}\right) = \cos^{-1}(0) = \boxed{90^\circ}$$

Between $\vec{b}$ and $\vec{c}$:

$$\vec{b} \cdot \vec{c} = \langle 1, -1, 1 \rangle \cdot \langle -2, 2, 1 \rangle = -2 - 2 + 1 = -3$$

$$\theta = \cos^{-1}\left(\frac{-3}{\sqrt{3}\ 3}\right) = \cos^{-1}\left(-\frac{1}{\sqrt{3}}\right) = \cancel{\boxed{-60^\circ}}$$

$$= \boxed{125.26^\circ}$$

11.3.31

$$\vec{u} = 3\hat{i} + 2\hat{j} + \hat{k}$$
$$\vec{v} = 2\hat{i} - \hat{k}$$
$$\vec{w} = \hat{i} + 5\hat{j} - 3\hat{k}$$

$$\text{proj}_{\vec{u}}\,\vec{w} = \left(\frac{\vec{w}\cdot\vec{u}}{\|\vec{u}\|^2}\right)\vec{u}$$

$$\vec{w}\cdot\vec{u} = 3+10-3 = 10$$

$$\|\vec{u}\|^2 = 3^2+2^2+1^2 = 14$$

$$\Rightarrow \quad \left(\frac{10}{14}\right)\langle 3, 2, 1\rangle$$

$$= \boxed{\left\langle \frac{15}{7}, \frac{10}{7}, \frac{5}{7} \right\rangle}$$

11.3.66

Find the equation of the plane having the given normal vector $\vec{n}$ and passing through the given point P.

$$\vec{n} = 3\hat{i} - 2\hat{j} - 1\hat{k}$$

$$P(-2, -3, 4)$$

The equation of the plane will be:

$$3(x-(-2)) - 2(y-(-3)) - 1(z-4) = 0$$

$$\Rightarrow 3x - 2y - z = -12 + 6 - 4$$

$$\Rightarrow \boxed{3x - 2y - z = -10}$$

11.3.74

Find the distance from (2, 6, 3) to the plane $-3x+2y+z=9$.

$$L = \frac{|-3(2)+2(6)+1(3)-9|}{\sqrt{(-3)^2+2^2+1^2}}$$

$$= \frac{|-6+12+3-9|}{\sqrt{9+4+1}}$$

$$= \boxed{0}$$

Note:

$-3(2)+2(6)+3 = -6+12+3=9.$

So, (2, 6, 3) is a point <u>on</u> the plane $-3x+2y+z=9$, and so the distance is 0.

11.4.1

$\vec{a} = -3\hat{i} + 2\hat{j} - 2\hat{k} \qquad \vec{b} = -\hat{i} + 2\hat{j} - 4\hat{k}$

$\vec{c} = 7\hat{i} + 3\hat{j} - 4\hat{k}$

a) $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 2 & -2 \\ -1 & 2 & -4 \end{vmatrix}$ $(-8-(-4))\hat{i} + (2-12)\hat{j} + (-6-(-2))\hat{k}$

= [illegible]

$= \cancel{-20\hat{i} - 10\hat{j} - 4\hat{k}}$

$= \boxed{-4\hat{i} - 10\hat{j} - 4\hat{k}}$

b) $\vec{a} \times (\vec{b} + \vec{c}) \qquad \vec{b} + \vec{c} = 6\hat{i} + 5\hat{j} - 8\hat{k}$

$\vec{a} \times (\vec{b} + \vec{c}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 2 & -2 \\ 6 & 5 & -8 \end{vmatrix} = (-16-(-10))\hat{i}$

$+ (-12-24)\hat{j}$

$+ (-3(5)-12)\hat{k}$

$= \boxed{-6\hat{i} - 36\hat{j} - 27\hat{k}}$

c) $\vec{a} \cdot (\vec{b} + \vec{c}) = \langle -3, 2, -2 \rangle \cdot \langle 6, 5, -8 \rangle$

$= -18 + 10 + 16 = \boxed{8}$

d) $\vec{b} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 2 & -4 \\ 7 & 3 & -4 \end{vmatrix} = (-8-(-12))\hat{i} + (-28-4)\hat{j} + (-3-14)\hat{k}$

$= 4\hat{i} - 32\hat{j} - 17\hat{k}$

$\vec{a} \times (\vec{b} \times \vec{c}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 2 & -2 \\ 4 & -32 & -17 \end{vmatrix} = (-34-64)\hat{i} + (-8-51)\hat{j} + (96-8)\hat{k}$

= [illegible]

$\boxed{-98\hat{i} - 59\hat{j} + 88\hat{k}}$

11.4.3

Find all vectors perpendicular to both of the vectors

$$\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$$
$$\vec{b} = -2\hat{i} + 2\hat{j}\cancel{4} - 4\hat{k}$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ -2 & 2 & -4 \end{vmatrix} = (-8-6)\hat{i} + (-6-(-4))\hat{j} + (2-(-4))\hat{k}$$

$$= -14\hat{i} - 2\hat{j} + 6\hat{k}$$

So, the set of all vectors perpendicular to both $\vec{a}$ and $\vec{b}$ is:

$$c\langle -14, -2, 6 \rangle \qquad c \in \mathbb{R}.$$

11.4.12

Find the equation of the plane through the points:

$(1,1,2)$, $(0,0,1)$, and $(-2,-3,0)$.

$$\vec{a} = \langle 1-0, 1-0, 2-1\rangle = \langle 1,1,1\rangle$$

$$\vec{b} = \langle -2-0, -3-0, 0-1\rangle = \langle -2,-3,-1\rangle$$

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ -2 & -3 & -1 \end{vmatrix} = (-1-(-3))\hat{i} + (-2-(-1))\hat{j} + (-3-(-2))\hat{k}$$

$$= 2\hat{i} - \hat{j} - \hat{k}$$

$$2(x-0) + (-1)(y-0) + (-1)(z-1) = 0$$

$$\Rightarrow \boxed{2x - y - z = -1}$$

11.4.15

Find the equation of the plane through (2, 5, 1) that is parallel to the plane $x - y + 2z = 4$.

$$1(x-2) + (-1)(y-5) + 2(z-1) = 0$$

$$x - 2 - y + 5 + 2z - 2 = 0$$

$$\Rightarrow x - y + 2z + 1 = 0$$

$$\Rightarrow \boxed{x - y + 2z = -1}$$