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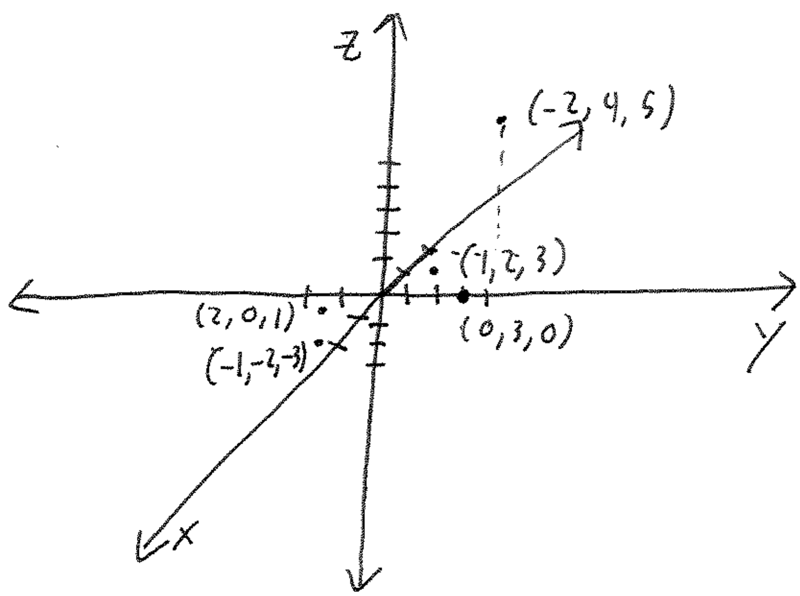
Math 2210
Assignment #1
Solutions

11.1.1:

Plot the points whose coordinates are $(1,2,3)$, $(2,0,1)$, $(-2,4,5)$, $(0,3,0)$, and $(-1,-2,-3)$.



11.1.7

Show that (2, 1, 6), (4, 7, 9) and (8, 5, -6) are vertices of an equilateral triangle.

$$P_1 = (2, 1, 6)$$
$$P_2 = (4, 7, 9)$$
$$P_3 = (8, 5, -6)$$

$$\|\overrightarrow{P_1P_2}\| = \sqrt{(4-2)^2 + (7-1)^2 + (9-\cancel{8})^2}$$
$$= \sqrt{2^2 + 6^2 + 3^2}$$
$$= \sqrt{49} = 7$$

$$\|\overline{P_1P_3}\| = \sqrt{(8-2)^2 + (5-1)^2 + (-6-6)^2}$$
$$= \sqrt{6^2 + 4^2 + (-12)^2}$$
$$= \sqrt{36 + 16 + 144}$$
$$= \sqrt{196} = 14$$

$$\|\overline{P_2P_3}\| = \sqrt{(8-4)^2 + (5-7)^2 + (-6-9)^2}$$
$$= \sqrt{4^2 + (-2)^2 + (-15)^2}$$
$$= \sqrt{16 + 4 + 225}$$
$$= \sqrt{245}$$

Now,

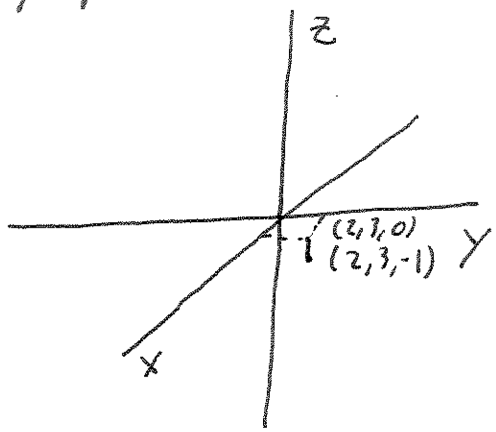
$$\|\overline{P_1P_2}\|^2 + \|\overline{P_1P_3}\|^2 = 7^2 + 14^2 = 245 = \left(\sqrt{245}\right)^2 = \|\overline{P_2P_3}\|^2$$

So, the triangle formed by P_1, P_2, and P_3 satisfies the Pythagorean theorem, and is therefore a right triangle.

1.1.8

Find the distance from $(2,3,-1)$ to

a) the xy-plane

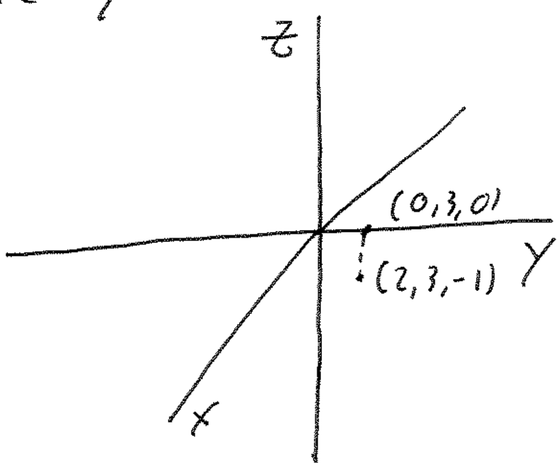


Distance =

$\|(2,3,-1)-(2,3,0)\|$

$= \boxed{1}$

b) the y-axis



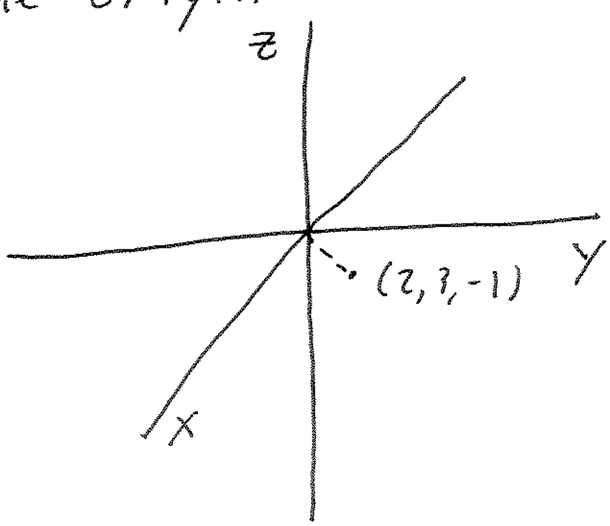
Distance =

$\|(2,3,-1)-(0,3,0)\|$

$= \sqrt{2^2+0^2+(-1)^2}$

$= \boxed{\sqrt{5}}$

c) the origin



Distance =

$\|(2,3,-1)-(0,0,0)\|$

$= \sqrt{2^2+3^2+(-1)^2}$

$= \boxed{\sqrt{14}}$

11.1.16

Complete the square to find the center and radius of the sphere whose equation is:

$$x^2+y^2+z^2+8x-4y-22z+77=0$$

$$\Rightarrow (x+4)^2+(y-2)^2+(z-11)^2-64=0$$

$$\Rightarrow (x+4)^2+(y-2)^2+(z-11)^2=8^2$$

So, center $=(-4,2,11)$

radius $=8$

11.1.31

Find the arc length of the curve:

$x = 2\cos t \quad y = 2\sin t \quad z = 3t$

$-\pi \le t \le \pi$

$\frac{dx}{dt} = -2\sin t \quad \frac{dy}{dt} = 2\cos t \quad \frac{dz}{dt} = 3$

$$L = \int_{-\pi}^{\pi} \sqrt{(-2\sin t)^2 + (2\cos t)^2 + 3^2}\, dt$$

$$= \int_{-\pi}^{\pi} \sqrt{4\sin^2 t + 4\cos^2 t + 9}\, dt$$

$$= \int_{-\pi}^{\pi} \sqrt{4+9}\, dt = \int_{-\pi}^{\pi} \sqrt{13}\, dt$$

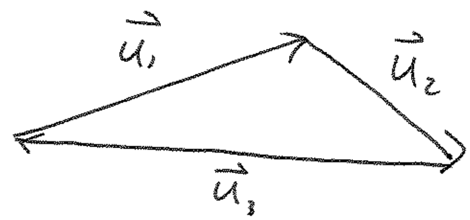
$$= \sqrt{13}\, t \Big|_{-\pi}^{\pi} = \sqrt{13}\,(\pi - (\pi))$$

$$= \boxed{2\sqrt{13}\,\pi}$$

11.2.4

Draw the vector $\vec{w}$:

$$\vec{w} = \vec{u}_1 + \vec{u}_2 + \vec{u}_3$$



$$\boxed{\vec{w} = \cdot}$$

11.2.11

For the two-dimensional vectors $\vec{u}$ and $\vec{v}$ find $\vec{u}+\vec{v}$, $\vec{u}-\vec{v}$, $\|\vec{u}\|$, and $\|\vec{v}\|$.

$$\vec{u} = \langle 12, 12 \rangle \qquad \vec{v} = \langle -2, 2 \rangle$$

$$\vec{u}+\vec{v} = \langle 12+(-2), 12+2 \rangle = \boxed{\langle 10, 14 \rangle}$$

$$\vec{u}-\vec{v} = \langle 12-(-2), 12-2 \rangle = \boxed{\langle 14, 10 \rangle}$$

$$\|\vec{u}\| = \sqrt{12^2+12^2} = \boxed{12\sqrt{2}}$$

$$\|\vec{v}\| = \sqrt{(-2)^2+2^2} = \boxed{2\sqrt{2}}$$

11.2.15

For the vectors $\vec{u}$ and $\vec{v}$ calculate $\vec{u}+\vec{v}$, $\vec{u}-\vec{v}$, $\|\vec{u}\|$, and $\|\vec{v}\|$

$$\vec{u} = \langle 1, 0, 1 \rangle$$
$$\vec{v} = \langle -5, 0, 0 \rangle$$

$$\vec{u} + \vec{v} = \langle 1+(-5), 0+0, 1+0 \rangle = \langle -4, 0, 1 \rangle$$

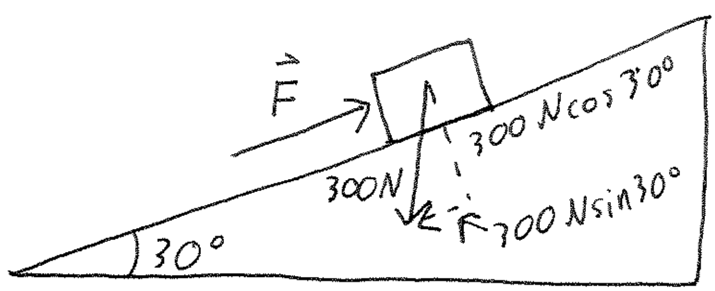
$$\vec{u} - \vec{v} = \langle 1, 0, 1 \rangle - \langle -5, 0, 0 \rangle = \langle 6, 0, 1 \rangle$$

$$\|\vec{u}\| = \sqrt{1^2 + 0^2 + 1^2} = \boxed{\sqrt{2}}$$

$$\|\vec{v}\| = \sqrt{(-5)^2 + 0^2 + 0^2} = \boxed{5}$$

11-2-19

A 300-Newton weight rests on a smooth (friction negligible) inclined plane that makes an angle of 30° with the horizontal. What force parallel to the plane will just keep the weight from sliding down the plane?

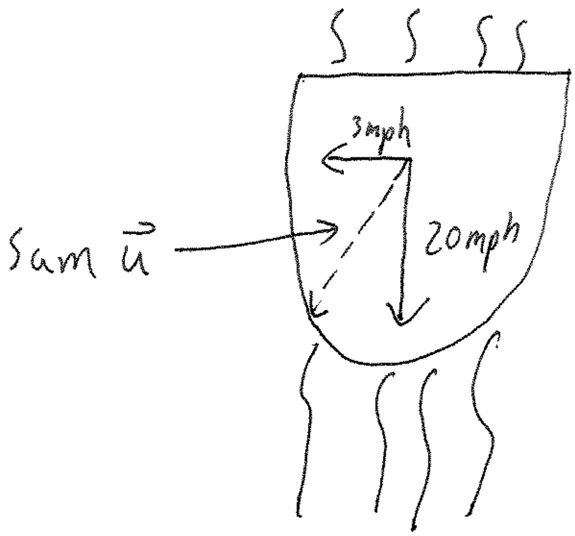


For the forces down the plane to balance we must have

$$\vec{F} = 300\,N \sin(30°)$$
$$= \boxed{150\,N \text{ up the plane}}$$

11.2.22

A ship is sailing due South at 20 m.p.h. A man walks west across the deck at 3 m.p.h. What are the magnitude and direction of his velocity relative to the surface of the water?



$$\|\vec{u}\| = \sqrt{3^2 + 20^2} = \sqrt{409} \approx \boxed{20.22 \text{ mph}}$$

$$\theta = \tan^{-1}\left(\frac{3}{20}\right) = \boxed{8.5^\circ \text{ West of South}}$$

So, at 20.22 mph 8.5° West of South.