Math 2210 - Assignment 8

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1 Sections 12.9 through 13.1

1.1 Section 12.9

12.9.1 Find the minimum of:

$$f(x, y) = x^2 + y^2$$

subject to the constraint:

$$g(x, y) = xy - 3 = 0$$

12.9.4 Find the minimum of:

$$f(x, y) = x^2 + 4xy + y^2$$

subject to the constraint:

$$x - y - 6 = 0.$$

12.9.6 Find the minimum of:

$$f(x, y, z) = 4x - 2y + 3z$$

subject to the constraint:

$$2x^2 + y^2 - 3z = 0.$$

12.9.8 Find the minimum distance between the origin and the plane:

$$x + 3y - 2z = 4$$

12.9.11 Find the maximum volume of a closed rectangular box with faces parallel to the coordinate planes inscribed in the ellipsoid:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

1.2 Section 13.1

13.1.1 Let $R = \{(x, y) : 1 \leq x \leq 4, 0 \leq y \leq 2\}$. Evaluate $\int\int_R f(x, y)dA$, where f is the function:

$$f(x, y) = \begin{cases} 2 & 1 \leq x < 3, 0 \leq y \leq 2 \\ 3 & 3 \leq x \leq 4, 0 \leq y \leq 2 \end{cases}$$

13.1.4 Let $R = \{(x, y) : 1 \leq x \leq 4, 0 \leq y \leq 2\}$. Evaluate $\displaystyle\int\int_R f(x, y) dA$, where f is the function:

$$f(x, y) = \begin{cases} 2 & 1 \leq x \leq 4, 0 \leq y < 1 \\ 3 & 1 \leq x < 3, 1 \leq y \leq 2 \\ 1 & 3 \leq x \leq 4, 1 \leq y \leq 2 \end{cases}$$

13.1.6 Suppose that

$$R = \{(x, y) : 0 \leq x \leq 2, 0 \leq y \leq 2\}$$
$$R_1 = \{(x, y) : 0 \leq x \leq 2, 0 \leq y \leq 1\}$$
$$R_2 = \{(x, y) : 0 \leq x \leq 2, 1 \leq y \leq 2\}.$$

Suppose, in addition, that:

$$\int\int_R f(x, y)dA = 3, \int\int_R g(x, y)dA = 5, \text{ and } \int\int_{R_1} g(x, y)dA = 2.$$

Use the properties of integrals to evaluate the integral:

$$\int\int_R [2f(x, y) + 5g(x, y)]dA$$

13.1.8 Suppose that

$$R = \{(x, y) : 0 \leq x \leq 2, 0 \leq y \leq 2\}$$
$$R_1 = \{(x, y) : 0 \leq x \leq 2, 0 \leq y \leq 1\}$$
$$R_2 = \{(x, y) : 0 \leq x \leq 2, 1 \leq y \leq 2\}.$$

Suppose, in addition, that:

$$\int\int_R f(x,y)dA = 3,\ \int\int_R g(x,y)dA = 5, \text{ and } \int\int_{R_1} g(x,y)dA = 2.$$

Use the properties of integrals to evaluate the integral:

$$\int\int_{R_1} [2g(x,y) + 3]dA$$

13.1.11 Suppose

$$R = \{(x, y) : 0 \leq x \leq 6, 0 \leq y \leq 4\}$$

and P is the partition of R into six equal squares by the lines:

$$x = 2, x = 4, \text{ and } y = 2.$$

Approximate $\int\int_R f(x, y)dA$ by calculating the corresponding Riemann sum $\sum_{k=1}^{6} f(\overline{x}_k, \overline{y}_k)\Delta A_k$, assuming that $(\overline{x}_k, \overline{y}_k)$ are the centers of the six squares.

$$f(x, y) = x^2 + 2y^2$$