# Math 2210 - Assignment 6

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## 1 Sections 12.4 through 12.6

### 1.1 Section 12.4

**12.4.1** Find the gradient, $\bigtriangledown f(x,y)$, of the function $f(x,y)$:

$$f(x,y) = x^2y + 3xy$$

**12.4.3** Find the gradient, $\bigtriangledown f(x, y)$, of the function $f(x, y)$:

$$f(x, y) = xe^{xy}$$

**12.4.8** Find the gradient, $\bigtriangledown f(x, y, z)$, of the function $f(x, y, z)$:

$$f(x, y, z) = x^2y + y^2z + z^2x$$

**12.4.11** Find the gradient vector of the given function at the given point **p**. Then find the equation of the tangent plane at **p**.

$$f(x, y) = x^2y - xy^2, \mathbf{p} = (-2, 3)$$

**12.4.20** Find all points $(x, y)$ at which the tangent plane to the graph of $z = x^3$ is horizontal.

## 1.2 Section 12.5

**12.5.1** Find the directional derivative of $f$ at the point **p** in the direction of **a**:

$$f(x, y) = x^2 y; \mathbf{p} = (1, 2); \mathbf{a} = 3\mathbf{i} - 4\mathbf{j}.$$

**12.5.6** Find the directional derivative of $f$ at the point $\mathbf{p}$ in the direction of $\mathbf{a}$:

$$f(x, y) = e^{-xy}; \mathbf{p} = (1, -1); \mathbf{a} = -\mathbf{i} + \sqrt{3}\mathbf{j}.$$

**12.5.8** Find the directional derivative of $f$ at the point $\mathbf{p}$ in the direction of $\mathbf{a}$:

$$f(x, y, z) = x^2 + y^2 + z^2; \mathbf{p} = (1, -1, 2); \mathbf{a} = \sqrt{2}\mathbf{i} - \mathbf{j} - \mathbf{k}.$$

**12.5.14** In what direction **u** does $f(x, y) = \sin(3x - y)$ decrease most rapidly at $\mathbf{p} = (\pi/6, \pi/4)$.

**12.5.21** Find the gradient of $f(x, y, z) = \sin\sqrt{x^2 + y^2 + z^2}$. Show that the gradient always points directly toward the origin or directly away from the origin.

## 1.3 Section 12.6

**12.6.1** Find $dw/dt$ by using the chain rule. Express your final answer in terms of $t$.

$$w = x^2y^3; x = t^3, y = t^2.$$

**12.6.4** Find $dw/dt$ by using the chain rule. Express your final answer in terms of $t$.

$$w = \ln(x/y);\ x = \tan t,\ y = (\sec t)^2.$$

**12.6.7** Find $\partial w/\partial t$ by using the chain rule. Express your final answer in terms of $s$ and $t$.

$$w = x^2y; x = st, y = s - t.$$

**12.6.11** Find $\partial w/\partial t$ by using the chain rule. Express your final answer in terms of $s$ and $t$.

$$w = \sqrt{x^2 + y^2 + z^2};\ x = \cos(st),\ y = \sin(st),\ z = s^2 t.$$

**12.6.20** Sand is pouring onto a conical pile in such a way that at a certain instant the height is 100 inches and increasing at 3 inches per minute and the base radius is 40 inches and increasing at 2 inches per minute. How fast is the volume increasing at that instant?