Math 2210 - Assignment 5

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1 Sections 12.1 through 12.3

1.1 Section 12.1

12.1.1 Let $f(x, y) = x^2y + \sqrt{y}$. Find each value.

1. $f(2, 1)$

2. $f(3, 0)$

3. $f(1, 4)$

4. $f(a, a^4)$

5. $f(1/x, x^4)$

6. $f(2, -4)$

What is the natural domain for this function?

12.1.6 Find $F(f(t), g(t))$ if $F(x, y) = e^x + y^2$ and $f(t) = \ln t^2$, $g(t) = e^{t/2}$.

12.1.17 Sketch the level curve $z = k$ for the indicated values of k.

$$z = \frac{1}{2}(x^2 + y^2),\ k = 0, 2, 4, 6, 8.$$

12.1.27 Describe geometrically the domain of the function:

$$f(x, y, z) = \sqrt{x^2 + y^2 + z^2 - 16}.$$

12.1.33 Describe geometrically the level surfaces for the function:

$$f(x, y, z) = x^2 + y^2 + z^2; \, k > 0$$

1.2 Section 12.2

12.2.1 Find all the partial derivatives of the function:

$$f(x, y) = (2x - y)^4$$

12.2.5 Find all the partial derivatives of the function:

$$f(x, y) = e^y \sin x$$

12.2.13 Find all the partial derivatives of the function:

$$f(x, y) = y \cos x^2 + y^2$$

12.2.19 Verify that:

$$\frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y}$$

for the function:

$$f(x, y) = 3e^{2x} \cos y$$

12.2.34 A function of two variables that satisfies *Laplace's Equation*,

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$$

is said to be *harmonic*. Show that the function:

$$f(x, y) = \ln\left(4x^2 + 4y^2\right)$$

is harmonic.

1.3 Section 12.3

12.3.1 Find the limit or state that it does not exist:

$$\lim_{(x,y)\to(1,3)} (3x^2y - xy^3)$$

12.3.4 Find the limit or state that it does not exist:

$$\lim_{(x,y)\to(1,2)} \frac{x^3 - 3x^2y + 3xy^2 - y^3}{y - 2x^2}$$

12.3.11 Find the limit or state that it does not exist:

$$\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2+y^2}}$$

12.3.16 Find the limit or state that it does not exist:

$$\lim_{(x,y)\to(0,0)} \frac{xy^2}{x^2+y^4}$$

12.3.30 Sketch the set S and describe the boundary of the set. Finally, state whether the set is open, closed, or neither.

$$S = \{(x, y) : 1 < x \leq 4\}$$