Timed Practice Final 2

Math 1030 - Dylan Zwick's Class

Fall 2007

Name: Solutions

Savings Plan Formula

$$A = PMT\left(\frac{(1+\frac{APR}{n})^{nY}-1}{(\frac{APR}{n})}\right)$$

Loan Formula

$$PMT = \frac{P(\frac{APR}{n})}{\left(1-(1+\frac{APR}{n})^{-nY}\right)}$$

Part 1 (30 points) There are five questions and each question is worth 6 points.

1. You plan to travel to Hyrule and you are practicing how to deal with different measurement systems. Consider the following situation.

   Suppose healing potions are priced at 150 rupees per liter. What is the price of these potions in U.S. dollars per gallon if one U.S. dollar is worth 12 rupees and 1 gallon is 3.7854 liters.

   $$\left(\frac{150 \text{ rupees}}{1 \text{ liter}}\right)\left(\frac{\$1}{12 \text{ rupees}}\right)\left(\frac{3.7854 \text{ L}}{1 \text{ gallon}}\right) = \boxed{\$47.32/\text{gallon}}$$

2. If you deposit \$1000 today and you can get an APR of 12% compounded continuously, how much will you have in 50 years?

   $$A = \$1{,}000(1+.012)^{50} = \boxed{\$289{,}002.19}$$

3. Between the first midterm and the second midterm scores increased from an average of 34 points to an average of 42 points. What percentage change is this from the first to the second midterm?

   $$\%\text{ change} = \frac{42-34}{34} \times 100\% = \boxed{23.5\%}$$

4. A sphere has a surface area of $400\pi\ m^2$. What is the radius of the sphere? A 1/100 scale model of the sphere is created. What is the scale model's surface area, expressed in cm^2?

$SA = 4\pi r^2 = 400\pi\ m^2$

$\Rightarrow r^2 = 100\,m^2 \Rightarrow \boxed{r = 10m}$

Scale model radius $10m\left(\frac{1}{100}\right) = .1m = 10cm$

$\Rightarrow$ surface area is $4\pi(10cm)^2 = \boxed{400\pi\,cm^2}$

Note $400\pi \approx 1256.64$

5. Your castle is worth \$2,500,000. If the value of the castle is increasing at a rate of 1.8% per year, how much will your castle be worth 250 years from now?

$A = \$2,500,000 \times (1.018)^{250}$

$= \boxed{\$216,214,347.24}$

Does this situation represent a linear or exponential model? Why?

Exponential. The growth rate is proportional.

Part II (70 points) - There are 7 questions and each question is worth 10 points.

1. At the start of a great Saturday you watch a 2 hour Simpson's DVD. Then, starting at noon, you watch a Simpsons marathon on TV. Each episode is one half hour long, and throughout the day you keep track of the number of episodes you've watched. (So the DVD was 4 episodes.)

   (a) Identify the independent and dependent variables:

   Dependent = # of episodes watched

   Independent - time

   (b) Write a linear equation that describes the situation.

   If time 0 is at noon:

   $\boxed{E = 2t + 4}$ t in hours

   E is episodes watched

   (c) How long does it take for you to watch 15 episodes?

   $15 = 2t + 4$

   $\Rightarrow t = \frac{11}{2} = 5.5$

   So, 5.5 hours. (At 5:30 PM you'll have seen 15 episodes that day.)

2. A witch is mixing a potion ~~is~~ in a cauldron. The cauldron is in the shape of a cylinder with a radius of 1 foot, and a height of 3 feet. Troll's blood flows into the cylinder at a rate of 10 cm^3 per second. How many minutes will it take until the cauldron is full? 1 inch = 2.54 cm.

Volume of cauldron = $3ft \times \pi (1ft)^2 = 3\pi\, ft^3$

Volume in $cm^3 = (3\pi\, ft^3)\left(\frac{12\,in}{1\,ft}\right)^3\left(\frac{2.54\,cm}{1\,in}\right)^3 = 266,879.99\,cm^3$

$\approx 266,880\,cm^3$

Time until full $\frac{266,880\,cm^3}{10\,cm^3/s} = 26,688\,s\left(\frac{1\,min}{60\,s}\right) \approx \boxed{444.8\,min}$

3. A magic penny is such that when you put it under your bed, the next day you have two magic pennies. If you keep putting all the pennies under your bed each day, after the first day you have 2 pennies, after the second day you have 4, after the third day you have 8, etc...

(a) How long will it take for you to have 256 pennies?

$256 = 2^d$ $\quad d = \#$ of days.

$\Rightarrow \log_{10}(256) = d \log_{10}(2)$

$\Rightarrow d = \frac{\log_{10}(256)}{\log_{10}(2)} = \boxed{8 \text{ days}}$

(b) If there is initially 1 penny, how much money do you have in dollars in one year? (1 year = 365 days)

$A = 2^{(365)} = 7.5 \times 10^{109}\,¢$

$= \boxed{\$7.5 \times 10^{107}}$

So, well over a google dollars!

4. You take a potion of growth and you double in size, and then your size decreases at an exponential rate back down to your original size. Two hours after taking the potion you are 80% larger than normal.

   (a) What is your rate of decrease back to your original size?

$$A = A_0(1+r)^t \qquad A = .8A_0$$
$$\Rightarrow .8 = (1+r)^2 \qquad t = 2$$
$$\Rightarrow (.8)^{1/2} = 1+r \Rightarrow r = (.8)^{1/2} - 1 = \boxed{-.1056}$$

   (b) What is the half-life of your decay in size?

$$T_{1/2} = \frac{-\log_{10}(2)}{\log_{10}(1+r)} = \boxed{6.21 \text{ hours}}$$

5. A savings account pays an annual percentage rate of 6% compounded quarterly (4 times per year).

   (a) Find the annual percentage yield on this account.

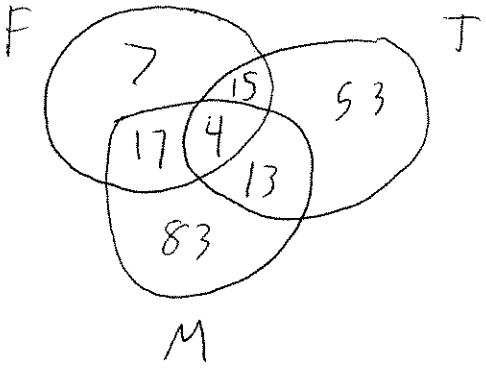
$$APY = \left(1 + \frac{.06}{4}\right)^4 - 1 = .06136$$
$$= \boxed{6.136\%}$$

   (b) You decide that you would like to make quarterly deposits to this account since you would like to have \$3,000,000 when you retire in 35 years. How much should your quarterly deposits be in order to accomplish your goal?

$$PMT = \frac{\$3,000,000}{\left(\frac{\left(1+\frac{.06}{4}\right)^{(4\times 35)} - 1}{\left(\frac{.06}{4}\right)}\right)} = \boxed{\$6,392.22}$$

6. In a population of 262 adventurers there are 113 fighters, 85 thieves, and 117 mages. Noreover, 4 were all three classes, 17 were fighter-mages, and 15 were mage-thieves, and 53 were just thieves.

(a) Draw a Venn diagram to illustrate this information. Use the symbols F, T, and M to represent the set of fighters, thieves, and mages, respectively, in the population.



$85-53-15-4=13$

$117-17-4-13=83$

$113-17-15-4=77$

(b) How many adventurers are just mages?

83

7. You have found that you are eligible for a 50 year house loan with annual interest rate (APR) of 7% compounded monthly.

(a) If you take out this loan for \$700,000, what will your monthly payment be?

$$PMT = \frac{\$700{,}000\left(\frac{.07}{12}\right)}{\left(1-\left(1+\frac{.07}{12}\right)^{-(12\times 50)}\right)} = \boxed{\$4{,}211.82}$$

(b) How much will you pay in interest (in \$ terms) over the life of the loan if you take out this loan for \$700,000?

$\$4{,}211.82 \times 12 \times 50 - \$700{,}000 = \boxed{\$1{,}827{,}091.51}$

(c) If you decide instead to get a 35-year loan at the same rate for the same amount, what would your monthly payment be and how much would you save (in dollars) in interest (if you decided to take a 35 year loan instead of 50 year loan).

$$PMT = \frac{\$700{,}000\left(\frac{.07}{12}\right)}{\left(1-\left(1+\frac{.07}{12}\right)^{-(12\times 35)}\right.}$$

$= \$4,471.99$

Interest on 35 year loan

$\$4,471.99 \times 12 \times 35 - \$700{,}000 = \boxed{\$1,178,237.69}$

Savings over 50 year loan

$\$1,827,091.51 - \$1,178,237.69$

$= \boxed{\$648,853.82}$