Timed Practice Final 1

Math 1030 - Dylan Zwick's Class

Fall 2007

Name: *Solutions*

Savings Plan Formula

$$A = PMT\left(\frac{(1 + \frac{APR}{n})^{nY} - 1}{(\frac{APR}{n})}\right)$$

Loan Formula

$$PMT = \frac{P(\frac{APR}{n})}{\left(1 - (1 + \frac{APR}{n})^{-nY}\right)}$$

Part 1 (30 points) There are five questions and each question is worth 6 points.

1. You plan to travel to Beirut and you are practicing how to deal with different measurement systems. Consider the following situation.

   Suppose dates are priced at 800 Lebanese Pounds (currency) per kilogram. What is the price of these dates in U.S. dollars per pound (weight) if one U.S. dollar is worth 1,512 Lebanese Pounds (currency) and 1 kilogram is 2.2 pounds (weight).

   $$\left(\frac{800 \text{ pounds}}{kg}\right)\left(\frac{\$ 1}{1,512 \text{ pounds}}\right)\left(\frac{1 kg}{2.2 lb}\right) = \boxed{\$.24 / lb}$$

2. If you deposit \$7000 today and you can get an APR of 6% compounded continuously, how much will you have in 19 years?

   $$A = \$7,000 \, e^{(.06 \times 19)}$$
   $$= \boxed{\$21,887.39}$$

3. Between July 2007 and August 2007 U.S. oil imports fell from 423,988 barrels to 421,565 barrels. What percentage change is this from July to August?

   $$\% \text{ change} = \frac{421,565 - 423,988}{423,988} \times 100\%$$
   $$= \boxed{-.571\%}$$

4. A cylinder has a surface area of $628.3 in^2$ and a volume of $1178.1 in^3$. A scale model of the cylinder is created that has a width of $15 in$. What will be the surface area and the volume of the model?

Surface area of cylinder $= 2\pi r h$

Volume of cylinder $= \pi r^2 h$

$$\frac{Volume}{SA} = \frac{\pi r^2 h}{2\pi r h} = \frac{r}{2} = \frac{1178.1 in^3}{628.3 in^2} = 1.875 in$$

$\Rightarrow r = 3.750 in$. So, the width of the cylinder is $2r = \boxed{7.5 in}$ The model is a doubling in scale.

The surface area of the model is: $628.3 in^2 \times 2^2 = \boxed{2,513.2 in^2}$, Volume of model is $1178.1 in^3 \times 2^3 = \boxed{9,424.8 in^3}$

5. Your apartment is worth $90,000. If the value of the apartment is increasing at a rate of 2.5% per year, how much will your apartment be worth 20 years from now?

$$A = P(1+r)^t = \$90,000(1+.025)^{20}$$
$$= \boxed{\$147,475.48}$$

Does this situation represent a linear or exponential model? Why?

Exponential. The rate of increase is proportional, not constant.

Part II (70 points) - There are 7 questions and each question is worth 10 points.

1. On Halloween you set of trick-or-treating with 5 candies in your bag. For each hour of trick-or-treating you receive another 10 candies.

   (a) Identify the independent and dependent variables:

   $$Independent - \cancel{to} \ Time$$
   $$Dependent - Candies \ in \ bag.$$

   (b) Write a linear equation that describes the situation.

   $$C = 10t + 5$$

   (c) How long does it take for you to have 27 candies?

   $$27 = 10t + 5$$

   $$\Rightarrow 22 = 10t \Rightarrow t = \frac{22}{10} = \boxed{2.2 \ hours}$$

2. An empty water tank is in the shape of a cylinder with a radius of 1 meter and a height of 2 meters. Water flows into the tank at a rate of 3 cubic yeards per second. How many minutes will it take until the tank is full? 1 meter = 1.094 yards.

$$\text{Volume of tank} = \pi r^2 h = \pi (1m)^2 (2m) = 2\pi \, m^3$$

$$\text{In yards} \quad 2\pi \, m^3 \left(\frac{1.094 \, yd}{1m} \right)^3 = 8.2268 \, yd^3$$

$$\text{Time till full} = 8.2268 \, yd^3 \left(\frac{1 \, sec}{3 \, yd^3} \right) = 2.742 \, sec$$

$$\text{In minutes} \quad 2.742 \, sec \left(\frac{1 \, min}{60 \, sec} \right) = \boxed{.0457 \, min}$$

3. A bacteria culture has a doubling time of 2 hours.

(a) How long will it take for the bacteria population to triple in size?

$$P = P_0 \, 2^{(t/T_2)} \qquad P = 3P_0 \quad T_2 = 2 \, hours$$

$$\Rightarrow 3P_0 = P_0 \, 2^{(t/2)} \Rightarrow 3 = 2^{t/2}$$

$$\Rightarrow \log_{10} 3 = \frac{t}{2} \log_{10} 2 \Rightarrow t = \frac{2 \log_{10} 3}{\log_{10} 2} = \boxed{3.17 \, hours}$$

(b) If there are initially 2 bacteria, find the population after 72 hours.

$$P = P_0 \, 2^{(t/T_2)} = 2 \times 2^{(72 \, hours / 2 \, hours)}$$

$$= \boxed{137,438,953,472 \, bacteria}$$

5

4. You have a blood alcohol content at 10:00 PM of .07. A brethalizer at 2:00 AM says you have a blood alcohol content of .05.

(a) Assuming the blood alcohol content decays exponentially, what is its rate of decrease?

$$C(t) = C_0 (1+r)^t$$

$t = 4 \text{ hours}$
$C(4) = .05$
$C(0) = .07$

$$\Rightarrow .05 = .07(1+r)^4$$

$$\Rightarrow \left(\frac{.05}{.07}\right)^{1/4} = 1+r \Rightarrow r = \left(\frac{.05}{.07}\right)^{1/4} - 1 = \boxed{-.0807}$$

(b) What is the exact half-life of the alcohol in your bloodstream?

$$T_{1/2} = \frac{-\log_{10}(2)}{\log_{10}(1+r)} = \boxed{8.24 \text{ hours}}$$

5. A savings account pays an annual percentage rate of 4% compounded monthly.

(a) Find the annual percentage yield on this account.

$$APY = \left(1.\frac{.04}{12}\right)^{12} - 1 = .04074$$

$$= \boxed{4.074 \%}$$

(b) You decide that you would like to make regular monthly deposits to this account since you would like to have $750.000 when you retire in 35 years. How much should your monthly deposits be in order to accomplish your goal?

$$PMT = \frac{A}{\left(\frac{\left(1+\frac{APR}{n}\right)^{nY} - 1}{\left(\frac{APR}{n}\right)}\right)} = \boxed{\$ 820.81}$$

6. Of the 256 ~~206~~ movies that were rented today from a Blockbuster store 112 were comedies, 80 were dramas, 64 were action. Moreover, 3 were all three types, 15 were drama/comedies, 21 were action/comedies, and 51 were just dramas.

(a) Draw a Venn diagram to illustrate this information. Use the symbols D, C, and A to represent the set of drama, comedy and action movies, respectively, that people rented.

D

$$80-51-15-3=11$$
$$64-21-3-11=29$$
$$112-21-3-15=73$$

51   15   73
11   3
21
29

A
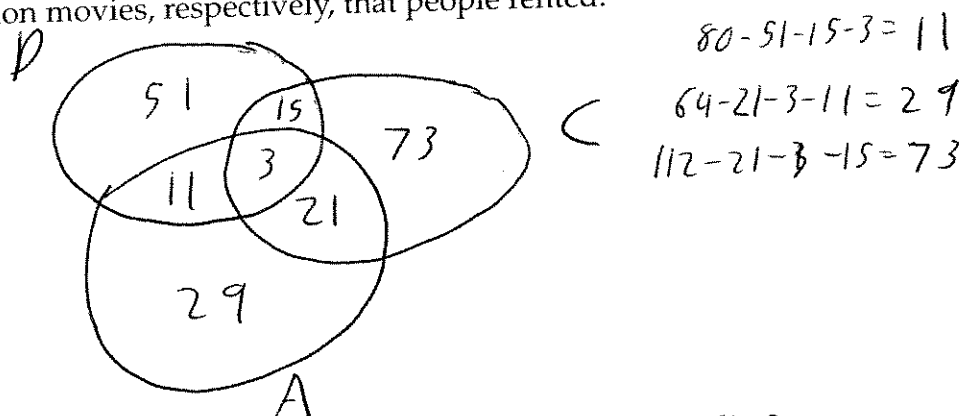
(b) How many people rented movies that were just comedies?

$$73$$

7. You have found that you are eligible for a 35 year house loan with annual interest rate (APR) of 6.75% compounded monthly.

(a) If you take out this loan for $450,000, what will your monthly payment be?

$$PMT = \frac{\$450,000\left(\frac{.0675}{12}\right)}{\left(1-\left(1+\frac{.0675}{12}\right)^{-(12\times35)}\right)} = \cancel{\$2,312.04}$$

$$= \boxed{\$2,716.37}$$

7

(b) How much will you pay in interest (in $ terms) over the life of the loan if you take out this loan for $450,000?

$$\$ \; 2{,}746.37 \times 12 \times 35 - \$ \, 450{,}000$$

$$= \; \cancel{\$ 54{,}055.40} \; = \; \boxed{\$ \, 724{,}475.40}$$

(c) If you decide instead to get a 15-year loan at the same rate for the same amount, what would your monthly payment be and how much would you save (in dollars) in interest (if you decided to take a 15 year loan instead of 35 year loan).

On 15 year loan,

$$PMT = \frac{\$450{,}000 \left(\frac{.0675}{12} \right)}{\left(1 - \left(1 + \frac{.0675}{12} \right)^{-(12 \times 15)} \right)}$$

$$= \boxed{\$ \, 3{,}982.04 \text{ /month}}$$

Total Interest:

$$\$ 3{,}982.04 \times 12 \times 15 - \$ 450{,}000$$

$$= \boxed{\$ \, 266{,}776.66}$$

Savings in interest over 35 year loan:

$$\$ 724{,}475.40 - \$ 266{,}776.66$$

$$= \boxed{\$ \, 457{,}698.74}$$