

Take Home Practice Final 2

Math 1030 - Dylan Zwick's Class

Fall 2007

Name: Solutions

Savings Plan Formula

$$A = PMT \left(\frac{(1 + \frac{APR}{n})^{nY} - 1}{(\frac{APR}{n})} \right)$$

Loan Formula

$$PMT = \frac{P(\frac{APR}{n})}{(1 - (1 + \frac{APR}{n})^{-nY})}$$

Part 1 (30 points) There are five questions and each question is worth 6 points.

1. You plan to travel to Paris and you are practicing how to deal with different measurement systems. Consider the following situation.

Suppose foie gras costs .2 Euros per gram? What is this price in US dollars per pound if 1.4738 US dollars is 1 Euro and 1 kilogram is 2.2 pounds?

$$\left(\frac{.2 \text{ Euros}}{\text{gram}} \right) \left(\frac{\$1.4738}{1 \text{ Euro}} \right) \left(\frac{1,000 \text{ grams}}{1 \text{ kg}} \right) \left(\frac{1 \text{ kg}}{2.2 \text{ lbs}} \right) = \boxed{\$133.98/\text{lb}}$$

2. If you deposit \$2,500 today and you can get an APR of 3% compounded continuously, how much will you have in 29 years?

$$A = (\$2,500) e^{(.03 \times 29)} = \boxed{\$5,967.28}$$

3. The population of Utah increased by 14.2% from 2000 to 2006. If the population was 2,233,169 in 2000, what is the approximate population in 2006?

$$2,233,169 \times (1 + .142) \approx \boxed{2,550,279}$$

4. A $1/5$ scale model of a triangle is being constructed. The triangle will be scaled down so that the scale model's base has a length of 5 cm. If the area enclosed by the original full scale triangle is 250 cm^2 what is the area enclosed by the scaled down model triangle? What was the height from the base of the original triangle?

Base of original triangle is $5 \times$ the length of the scaled down base, so $5 \times 5 \text{ cm} = 25 \text{ cm}$
 Area of original triangle is $\frac{1}{2} b \times h = 250 \text{ cm}^2$
 $\Rightarrow h = \frac{500 \text{ cm}^2}{25 \text{ cm}} = \boxed{20 \text{ cm}}$ The area of the scaled down model will

5. Your house is worth \$350,000. If the value of the house is increasing at a rate of 5.5% per year, how much will your house be worth in 10 years?

$$A = \$350,000 (1 + 0.055)^{10}$$

$$= \boxed{\$597,850.96}$$

$$250 \text{ cm}^2 \left(\frac{1}{5}\right)^2$$

$$= \boxed{10 \text{ cm}^2}$$

Does this situation represent a linear or exponential model? Why?

Exponential. The rate of increase is proportional, not ~~const~~ a constant value.

Part II (70 points) - There are 7 questions and each question is worth 10 points.

1. During the rainy season it rains 27 inches per day. On the first day of a month during the rainy season you start recording rainfall at 0 inches, and at the end of each day you record the total rainfall up to that point for the month.

(a) Identify the independent and dependent variables:

Independent - Days
Dependent - Rainfall

(b) Write a linear equation that describes the situation.

$$R = 27d + 0$$

(c) How long does it take before you record 125 inches of rain?

$$125 = 27d + 0$$

$$\Rightarrow d = \frac{125}{27} \approx \boxed{4.6 \text{ days}}$$

2. An empty water tank is in the shape of a cube with a side length of 1.5 meters. Water flows into the tank at a rate of 3 cubic yards per second. How many minutes will it take until the tank is full? 1 meter = 1.094 yards.

$$\text{Volume of cube} = (1.5\text{m})^3 = 3.375\text{m}^3$$

$$\text{Volume of cube in yards} = 3.375\text{m}^3 \left(\frac{1.094\text{yd}}{1\text{m}} \right)^3 = 4.419\text{yd}^3$$

$$\text{time} = \frac{4.419\text{yd}^3}{3\text{yd}^3/\text{sec}} = 1.473\text{sec} \left(\frac{1\text{min}}{60\text{sec}} \right) = \boxed{0.246\text{min}}$$

3. A community of rabbits has a doubling time of 15 months.

- (a) How long will it take for the rabbit population to increase in size by five times?

$$P = 5P_0$$

$$5P_0 = P_0 2^{(t/15\text{ months})} \Rightarrow 5 = 2^{(t/15\text{ months})}$$

$$\Rightarrow \log_{10} 5 = \left(\frac{t}{15\text{ months}} \right) \log_{10} (2) \Rightarrow t = 15\text{ months} \left(\frac{\log_{10}(5)}{\log_{10}(2)} \right) = \boxed{34.8\text{ months}}$$

- (b) If there are initially 100 rabbits ~~mice~~, find the population after 7 years.

$$7\text{ years} = 84\text{ months}$$

$$P = 100 \times 2^{(84/15)} = \boxed{4,850\text{ rabbits}}$$

4. After an atomic bomb goes off you measure 200 mg of a radioactive isotope in a 1 kg soil sample. 5 hours later you make the same measurement on the same soil sample and find 140 mg of the radioactive isotope.

- (a) Assuming the isotope decays exponentially, what is the rate of decrease of the radioactive isotope in the sample?

$$P = P_0 (1+r)^t \quad P = 140 \text{ mg} \quad P_0 = 200 \text{ mg} \quad t = 5 \text{ hours}$$

$$\Rightarrow \frac{140 \text{ mg}}{200 \text{ mg}} = 200 \text{ mg} (1+r)^5 \Rightarrow \frac{140 \text{ mg}}{200 \text{ mg}}^{1/5} - 1 = r$$

$$r = -0.06885$$

- (b) What is the exact half-life of the isotope?

$$T_{1/2} = \frac{-\log_{10}(2)}{\log_{10}(1+r)} = \frac{-\log_{10}(2)}{\log_{10}(.93115)} = 9.717 \text{ hours}$$

5. A savings account pays an annual percentage rate of 3.75% compounded daily.

- (a) Find the annual percentage yield on this account.

$$APY = \left(1 + \frac{.0375}{365}\right)^{365} - 1 = .0382$$

$$= 3.82\%$$

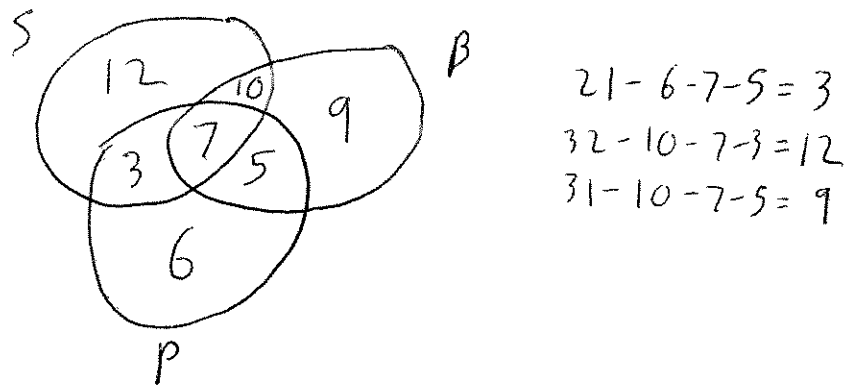
- (b) You decide that you would like to make regular *daily* deposits to this account since you would like to have \$2,000,000 when you retire in 45 years. How much should your daily deposits be in order to accomplish your goal?

~~$$PMT = \frac{\$2,000,000 \left(\frac{.0375}{365}\right)}{\left(1 - \left(1 + \frac{.0375}{365}\right)^{-(365 \times 45)}\right)} = \$252.12$$~~

$$PMT = \frac{\$2,000,000}{\left(\frac{\left(1 + \frac{.0375}{365}\right)^{(365 \times 45)} - 1}{\left(\frac{.0375}{365}\right)}\right)} = \$46.64$$

6. Of the 52 people on a train 32 like Shakira's music, 31 like Beyonce's music, and 21 like Pink's music. Moreover 7 like all 3, 5 like Beyonce and Pink only 10 like Beyonce and Shakira only, and 6 only like Pink.

- (a) Draw a Venn diagram to illustrate this information. Use the symbols S, B, and P to represent the set of people who like Shakira, Beyonce, and Pink, respectively.



- (b) How many people in the train only like Shakira?

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7. You have found that you are eligible for a 40 year house loan with annual interest rate (APR) of 6% compounded monthly.

- (a) If you take out this loan for \$300,000, what will your monthly payment be?

$$PMT = \frac{\$300,000 \left(\frac{0.06}{12} \right)}{\left(1 - \left(1 + \frac{0.06}{12} \right)^{-(12 \times 40)} \right)} = \$1,650.64$$

- (b) How much will you pay in interest (in \$ terms) over the life of the loan if you take out this loan for \$300,000?

$$\$1,650.64 \times 12 \times 40 - \$300,000 = \$492,307.64$$

- (c) If you decide instead to get a 25-year loan at the same rate for the same amount, what would your monthly payment be and how much would you save (in dollars) in interest (if you decided to take a 25 year loan instead of a 40 year loan).

For 25 year loan:

$$\begin{aligned}PMT &= \frac{\$300,000 \left(\frac{-0.6}{12}\right)}{\left(1 - \left(1 + \frac{-0.6}{12}\right)^{-(12 \times 25)}\right)} \\&= \$1,932.90\end{aligned}$$

Interest Paid:

$$\begin{aligned}&\$1,932.90 \times 12 \times 25 - \$300,000 \\&= \$279,871.26\end{aligned}$$

Savings over 40 year loan:

$$\begin{aligned}&\$492,307.64 - \$279,871.26 \\&= \boxed{\$212,436.38}\end{aligned}$$