Take Home Practice Final 1

Math 1030 - Dylan Zwick's Class

Fall 2007

Name: *Solutions*

Savings Plan Formula

$$A = PMT \left(\frac{(1 + \frac{APR}{n})^{nY} - 1}{(\frac{APR}{n})} \right)$$

Loan Formula

$$PMT = \frac{P(\frac{APR}{n})}{\left(1 - (1 + \frac{APR}{n})^{-nY}\right)}$$

Part 1 (30 points) There are five questions and each question is worth 6 points.

1. You plan to travel to London and you are practicing how to deal with different measurement systems. Consider the following situation.

   Suppose tickets on the Underground cost £1.5 per ticket. What is the price of 3 tickets in US dollars if there are 2.0631 £per dollar?

$$\left(3 \text{ tickets}\right)\left(\frac{£1.5}{\text{ticket}}\right)\left(\frac{1 \text{ dollar}}{£2.0631}\right) = \boxed{\$2.18}$$

2. If you deposit $1.500 today and you can get an APR of 5% compounded continuously, how much will you have in 31 years?

$$A = Pe^{APR \times Y} = \$1,500 \, e^{(.05 \times 31)}$$
$$= \boxed{\$7,067.21}$$

3. London (Reuters) 11/15/04 "U.S. light crude fell $1.80 to $45.60 a barrel, the lowest price since mid-September." By what percentage did the price change?

$$\% \text{ change} = \frac{\text{old} \quad \text{new value} - \text{old value}}{\text{old value}} \times 100\%$$

$$\text{old value} = \$45.60 + \$1.80 = \$47.40$$

$$\% \text{ change} = \frac{-\$1.80}{\$47.40} \times 100\% = \boxed{-3.80\%}$$

4. What is volume, in cubic feet, of a room that measures 3 meters by 4 meters by 3 meters? (There are 2.54 cm in an inch.)

$$V = h \times \ell \times w = 3m \times 4m \times 3m = 36\,m^3$$

In cubic feet:

$$36\,m^3 \left(\frac{1\,in}{2.54\,cm}\right)^3 \left(\frac{100cm}{1\,m}\right)^3 \left(\frac{1\,ft}{12\,in}\right)^3 = \boxed{1,271.33\,ft^3}$$

5. Your house is worth \$205,000. If the value of the house is increasing at a rate of 3.5% per year, how much will you rhouse be worth 20 years later?

$$A = P(1+r)^t = \$205,000\,(1+.035)^{20}$$

$$= \boxed{\$407,906.72}$$

Does this situation represent a linear or exponential model? Why?

Exponential. The rate of increase is proportional, not constant.

Part II (70 points) - There are 7 questions and each question is worth 10 points.

1. One morning, there were 3 inches of snow on the ground. Then the winter storm started and snow started accumulating at a constant rate of 5 inches every 2 hours.

   (a) Identify the independent and dependent variables:

   Independent = time
   Dependent = snow

   (b) Write a linear equation that describes the situation.

   $$S = 2.5t + 3$$

   S = inches of snow
   t = time in hours

   (c) How long did it take for the height of the snow to reach 21 inches?

   $$21 = 2.5t + 3$$
   $$\Rightarrow 18 = 2.5t$$
   $$\Rightarrow t = \frac{18}{2.5} = \boxed{7.2 \text{ hours}}$$

2. An empty water tank is in the shape of a sphere with a radius of 24 meters. Water flows into the tank at a rate of 18 cubic yeards per second. How many minutes will it take until the tank is full? 1 meter = 1.094 yards.

$$Volume \ of \ tank = \frac{4}{3} \pi r^3 = \frac{4}{3} \pi (24m)^3$$

$$= 57,905.84 \ m^3$$

$$Flow = \left(\frac{18 \ yd^3}{s}\right)\left(\frac{1 \ m}{1.094 \ yd}\right)^3 = 13.7474 \ m^3/s$$

$$Time \ until \ full = \frac{57,905.84 \ m^3}{13.7474 \ m^3/s} \approx 4212 \ s\left(\frac{1 \ min}{60 \ s}\right)$$

$$= \boxed{70.2 \ min}$$

3. A community of mice has a doubling time of 3 months.

(a) How long will it take for the mice population to triple in size?

$$P = P_0 \ 2^{(t/T_2)} \qquad For \ P = 3P_0 \quad T_c = 3 \ months$$

$$\Rightarrow 3P_0 = P_0 \ 2^{(t/3)} \Rightarrow 3 = 2^{(t/3)} = \frac{\log_{10} 3}{\log_{10} 2} = \frac{t}{3} \Rightarrow t = \frac{3 \log_{10} 3}{\log_{10} 2}$$

$$= \boxed{4.75 \ months}$$

(b) If there are initially 500 mice, find the population after 2 years.

$$2 \ years = 24 \ months$$

$$P = P_0 \ 2^{(t/T_2)}$$

$$\Rightarrow P = 500 \times 2^{\left(\frac{24 \ months}{3 \ months}\right)}$$

$$= 500 \times 2^8 = \boxed{128,000 \ mice}$$

5

4. You take 300 mg of a certain medication at 2 pm. A lab test done at 6 pm shows that you still have 120 mg of that medication left in your bloodstream.

(a) Assuming the medication decays exponentially, what is the rate of decrease of the medication in your bloodstream?

$$A = A_0 (1+r)^t \qquad t = 4 \text{ hours}$$

$$\Rightarrow 120mg = 300mg (1+r)^4 \qquad A = 120 \, mg$$
$$A_0 = 300 \, mg$$

$$\Rightarrow \left(\frac{120mg}{300mg}\right) = (1+r)^4 \Rightarrow \log\left(\frac{120mg}{300mg}\right)^{1/4} = 1+r \Rightarrow r = \left(\frac{120mg}{300mg}\right)^{1/4} - 1 = \boxed{-.205}$$

(b) What is the exact half-life of that medication in your bloodstream?

$$T_{1/2} = \frac{-\log_{10}(2)}{\log_{10}(1+r)} = \frac{-\log_{10}(2)}{\log_{10}(.795271)}$$

$$= \boxed{3.026 \text{ hours}}$$

5. A savings account pays an annual percentage rate of 2.5% compounded quarterly.

(a) Find the annual percentage yield on this account.

$$APY = \left(1 + \frac{.025}{4}\right)^{4} - 1 = \cancel{.02529} \; .025235$$
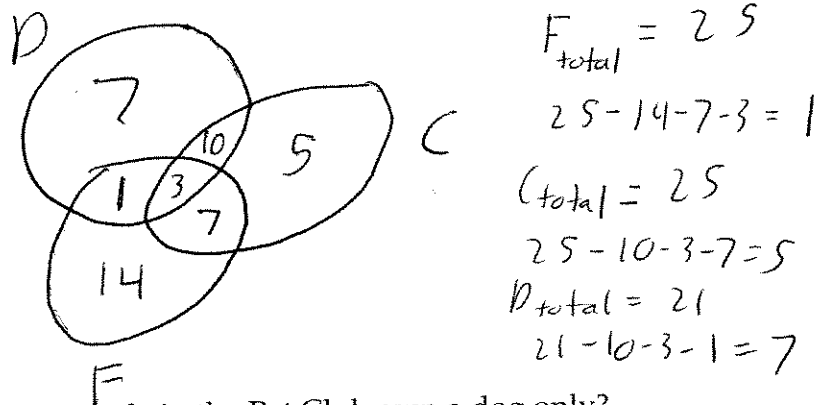
$$= \cancel{2.529} = \boxed{2.5235\%}$$

(b) You decide that you would like to make regular quarterly deposits to this account since you would like to have $750.000 when you retire in 40 years. How much should your quarterly deposits be in order to accomplish your goal?

$$PMT = \frac{A}{\left(\dfrac{\left(1 + \frac{APR}{n}\right)^{nY} - 1}{\left(\frac{APR}{n}\right)}\right)} = \frac{\$750,000}{\left(\dfrac{\left(1 + \frac{.025}{4}\right)^{4(40)} - 1}{\left(\frac{.025}{4}\right)}\right)}$$

$$= \boxed{\$27,414.92}$$

6. Of the 57 people in the Pet Club, 21 own a dog, 25 own a cat, 25 own a fish. Moreover, 3 own all three kinds of pets, 7 own a cat and a fish only, 10 own a cat and a dog only, and 14 own a fish only.

(a) Draw a Venn diagram to illustrate this information. Use the symbols D, C, and F to represent the set of people who own a dog, a cat, and a fish, respectively.

$$F_{total} = 25$$
$$25 - 14 - 7 - 3 = 1$$
$$C_{total} = 25$$
$$25 - 10 - 3 - 7 = 5$$
$$D_{total} = 21$$
$$21 - 10 - 3 - 1 = 7$$

(b) How many people in the Pet Club own a dog only?

$$7$$

7. You have found that you are eligible for a 30 year house loan with annual interest rate (APR) of 6.25% compounded monthly.

(a) If you take out htis loan for $200.000, what will your monthly payment be?

$$PMT = \frac{\$200,000 \left(\frac{.0625}{12} \right)}{\left(1 - \left(1 + \frac{.0625}{12} \right)^{-12 \cdot 30} \right)} = \boxed{\$1,231.43}$$

(b) How much will you pay in interest (in $ terms) over the life of the loan if you take out this loan for $200.000?

$$Interest = \$1,231.43 \times 12 \times 30 - \$200,000$$
$$= \boxed{\$243,316.38}$$

(c) If you decide instead to get a 20-year loan at the same rate for the same amount, what would your monthly payment be and how much would you save (in dollars) in interest (if you decided to take a 20 year loan instead of 30 year loan).

$$PMT = \frac{\$200,000 \left(\frac{.0625}{12}\right)}{\left(1 - \left(1 + \frac{.0625}{12}\right)^{-(12 \times 20)}\right)}$$

$$= \$1,461.86$$

Interest total

$$\$1,461.86 \times 12 \times 20 - \$200,000$$

$$= \$150,845.54$$

Total savings:

$$\$243,316.38 - \$150,845.54$$

$$= \boxed{\$92,470.84}$$