

Quiz 3 - Financial Math

Math 1030 - Dylan Zwick's Class

Fall 2007

Name: Solutions

Here are the formulas you may need:

Compound Interest Formula for Annual Compounding

$$A = P \times (1 + APR)^Y$$

Compound Interest Formula for Interest Paid n Times Per Year

$$A = P \left(1 + \frac{APR}{n} \right)^{nY}$$

Compound Interest Formula for Continuous Compounding

$$A = P \times e^{APR \times Y}$$

Savings Plan Formula (Regular Payments)

$$A = PMT \times \frac{\left[\left(1 + \frac{APR}{n} \right)^{nY} - 1 \right]}{\left(\frac{APR}{n} \right)}$$

Variable Definitions

A = Accumulated Amount

P = Starting Principal

APR = Annual Percentage Rate (express as a decimal)

Y = Number of Years

n = Number of Compounding Periods per Year

PMT = Regular Payment

1. Different Compounding Periods - 15 points

Suppose you invest \$2,500 in an account that generates compound interest with an APR of 5% for 30 years.

- (a) If your account compounds every year, how much money will you have in the account after 30 years? (3 points)

$$A = \$2,500 (1 + .05)^{30}$$
$$= \$10,804.86$$

- (b) If your account compounds every month, how much money will you have in the account after 30 years? (3 points)

$$A = \$2,500 \left(1 + \frac{.05}{12}\right)^{(12 \times 30)}$$
$$= \$11,169.36$$

- (c) If your account compounds continuously, how much money will you have in the account after 30 years? (3 points)

$$A = \$2,500 \times e^{(.05 \times 30)}$$
$$= \$11,204.22$$

- (d) What is the APY (annual percentage yield) of the account if it compounds continuously? (3 points)

$$\begin{aligned} APY &= (e^{(.05)} - 1) \times 100\% \\ &= \boxed{5.13\%} \end{aligned}$$

- (e) Suppose that instead of generating compound interest the account generated simple interest with an APR of 5%. How much money would you have in the account after 30 years? (3 points)

$$\begin{aligned} &\$2,500 + \$2,500 \times .05 \times 30 \\ &= \$2,500(1 + .05 \times 30) = \boxed{\$6,250} \end{aligned}$$

2. Savings Plan - 10 points

- (a) Suppose you start a retirement savings plan when you're 30 and want to retire with \$2,000,000 35 years later. If you invest in an account with a 4.5% APR compounded monthly, and you make monthly payments, how much money will you need to put away every month in order to retire with \$2,000,000? (5 points)

Note - The savings plan formula assumes compounding at the same rate as you make payments. This is the case for this problem.

$$\begin{aligned} PMT &= \frac{A \left(\frac{APR}{n} \right)}{\left[\left(1 + \frac{APR}{n} \right)^{nr} - 1 \right]} \\ &= \frac{\$2,000,000 \left(\frac{0.045}{12} \right)}{\left[\left(1 + \frac{0.045}{12} \right)^{(12 \times 35)} - 1 \right]} \\ &= \boxed{\$1,965.13} \end{aligned}$$

- (b) What is the total amount of money that you invest in this account over the 35 years? (2 points)

$$\begin{aligned} &\$1,965.13 \times 12 \times 35 \\ &= \boxed{\$829,356.56} \end{aligned}$$

- (c) Suppose that instead of putting away the money in monthly installments you invest one lump sum at the start and let it accumulate interest for 35 years in this account. How much money would you need in this lump sum in order to have \$2,000,000 after 35 years? (3 points)

$$\begin{aligned} P &= \frac{A}{\left(1 + \frac{APR}{n}\right)^{nY}} \\ &= \frac{\$2,000,000}{\left(1 + \frac{.045}{12}\right)^{(12 \times 35)}} \\ &= \boxed{\$415,236.49} \end{aligned}$$

3. Extra Credit - 5 points

- (a) Suppose you invest \$5,000 in a compound interest savings account with an APR of 5% that compounds annually. After Y number of years you have \$17,778.36 in your account. What is Y ? (Hint - logarithms) (4 points)

$$\begin{aligned} A &= P \times (1 + \text{APR})^Y \\ \Rightarrow \log A &= \log (P \times (1 + \text{APR})^Y) \\ \Rightarrow \log A &= \log P + Y \log (1 + \text{APR}) \\ \Rightarrow \frac{\log A - \log P}{\log (1 + \text{APR})} &= Y = \frac{\log (17,778.36) - \log (5000)}{\log (1.05)} \\ &= \boxed{26 \text{ years}} \end{aligned}$$

- (b) Name one character that appears in more than 3 of Shakespeare's plays. (1 point)

Many possible answers.
Example - Falstaff

More Extra Credit - Wednesday of next week is Halloween. We will have a costume contest the first 5 minutes of class. The people with the top 3 costumes, chosen democratically, will be awarded extra credit points on this quiz.

Best Costume - 3 points

Runner Up - 2 points

Second Runner Up - 1 point