Midterm 2 - Finances, Exponentials, and Modeling

Math 1030 - Dylan Zwick's Class

Fall 2007

Name: _

Financial Math Formulas

Compound Interest Formula for Annually Compounding Interest

 $A = P \times (1 + APR)^Y$

Compound Interest Formula for Interest Paid n Times Per Year

$$A = P \times \left(1 + \frac{APR}{n}\right)^{(nY)}$$

Compound Interest Formula for Continuous Compounding

 $A = P \times e^{APR \times Y}$

A = accumulated balance after Y years P = starting principal APR = annual percentage rate Y = number of years Savings Plan Formula (Regular Payments) Note - Assumes same times for payments and compounding periods, which will be the case on all problems in this midterm.

$$A = PMT \times \frac{\left[\left(1 + \frac{APR}{n}\right)^{(nY)} - 1\right]}{\left(\frac{APR}{n}\right)}$$

PMT = regular payment (deposit) amount n = number of payment periods per year Other Variables as Before

Exponential Formulas

Formula for Exponential Growth with the Doubling Time

$$y(t) = y_0 \times 2^{t/T_{double}}$$

Formula for Exponential Decay with the Doubling Time

$$y(t) = y_0 \times \left(\frac{1}{2}\right)^{t/T_{\frac{1}{2}}}$$

$$y(t)$$
 = amount after time t
 y_0 = initial amount (amount at t=0)
 T_{double} = doubling time
 $T_{\frac{1}{2}}$ = half-life
 T log 2

$$I_{double} \equiv \frac{1}{1+r}$$

r = growth rate (expressed as a decimal)

Logarithm Rules

$$\log_{10} (ab) = \log_{10} (a) + \log_{10} (b)$$

$$\log_{10} \left(\frac{a}{b}\right) = \log_{10} (a) - \log_{10} (b)$$

$$\log_{10} (a^{b}) = b \log_{10} (a)$$

$$\log_{a} (b) = \frac{\log_{10} (a)}{\log_{10} (b)}$$

1. Compound and Simple Interest (10 points)

Suppose you invest \$1000 in an account today, planning to use it for a year long world trip when you retire in 40 years. If you invest at a 5% APR interest, compounding annually, how much money will you have for your trip when you retire? (5 points)

How much money would you have if, instead of compounding annually, your interest compounded continuously? (4 points)

What is the APY for continuous compounding when the APR is 5%. (1 point)

2. Savings Plans (10 points)

Suppose you're smart and start saving for retirement when you're 20 years old. You invest \$1,000 each month for 45 years. If your APR is 4.5%, compounded monthly, how much money will you have when you retire? (4 points)

Suppose on the other hand that you would like to retire with \$2,000,000. Given the same set up as above (45 years, 4.5% APR, compounding monthly) what would your monthly payments need to be? (3 points)

Suppose that instead of making payments every month you put down one lump sum at the start. What would this lump sum need to be for you to have \$2,000,000 when you retire, using the same setup as the first two parts of this problem. (3 points)

3. Exponential Growth (10 points)

State whether each is a case of linear or exponential growth: (3 points)

- (a) A town whose population increases by 450 people every year.
- (b) A town whose population increases by 3% every year.
- (c) Compound interest.

If a town's population is 10,000 in 2007 and doubles every 5 years, what will its population be in 2022? (2 points)

Suppose one bacteria starts out in a bottle at 9:00 AM and the number of bacteria in the bottle double every minute through mitosis. If the bottle is full at 10:00 AM, at what time will the bottle be half full? (2 points)

If you have an account that generates 5% APR interest and is compounded annually, how long does it take for invested money to double as a result of interest? In other words, if you put \$1000 in the account and left it there, how long would it take for you to earn enough interest so that you've got \$2000 in the account? (3 points)

4. Functions and Logarithms (10 points)

Can a function have more than one input with the same output? (Yes/No) (1 point)

Can a function have more than one output with the same input? (Yes/No) (1 point)

The domain of a function is all possible inputs? (True/False) (1 point)

The range of a function is all possible outputs? (True/False) (1 point)

For the function $f(x) = x^2$ is the function one-to-one? (Yes/No) (1 point)

What are the domain and range of the function $f(x) = x^2$? (2 points)

If an odd function is periodic with period 3 and if f(1) = 2 what is f(-4)? (3 points)

5. Linear and Exponential Models (10 points)

Suppose you start trick-or-treating with 10 candies, and each hour you get another 30 candies. How many candies do you have after 3 hours? (2 points)

What is the equation for the line that passes through the points (3,5) and (5,9)? (5 points)

If there are 100 mG of an antibiotic in the bloodstream, and the drug decreases exponentially with a half-life of 12 hours, how much antibiotic will be in the bloodstream after 15 hours? (3 points)

6. Extra Credit (10 points)

Logarithms

Solve for *x* in the following equations:

(a) $47 = 3^x$ (2 points)

(b) $x = \log_3(7)$ (2 points)

(c) $99 = 12 \times 4^x$ (2 points)

For each of the following organizations or individuals indicate whether they are self declared Sunni or Shia Islam:

(3 points for 6/6, 2 points for 5/6, 1 point for 4/6)

- (a) Hezbollah -
- (b) Nouri al-Maliki -
- (c) Hamas -
- (d) Mahmoud Ahmadinejad -
- (e) Bashar al-Assad -
- (f) Pervez Musharraf -

Who is the President of Iraq? (1 point)