Diagnostic Test Solutions

Math 1030 - Dylan Zwick's Class

Fall 2007

1. Three kinds of apples are all mixed up in a basket. How many apples must you draw (without looking) from the basket to be sure of getting two apples of one kind?

Solution - You must draw 4 apples to be sure, as on the first three draws each apple could be different, but on the 4th you could not get a different type of apple.

2. There are 150 people in a class. If 80% of them are registered, how many are not registered?

Solution - A student is either registered or not, and if 80% are registered, then the remaining 20% are not. So, 20% are not registered, and 20% of 150 is:

150(.2) = 30

So, 30 students are not registered.

3. Express "three-fifths" as a fraction, a decimal, and as a percentage.

Solution

Fraction : $\frac{3}{5}$. Decimal : $\frac{3}{5} = .6$ Percentage : $\frac{3}{5} = .6 = 60\%$

4. Evaluate each of the following if a = 4, $b = \frac{2}{5}$, c = -6.

Solution

(a)
$$a(b+c) = 4(\frac{2}{5} + (-6)) = 4\frac{2}{5} - 24 = \frac{8}{5} - \frac{120}{5} = -\frac{112}{5}$$

(b) $ab + d\frac{a}{b} - c = (4)(\frac{2}{5}) + (-6)(\frac{4}{\frac{2}{5}}) - (-6) = \frac{8}{5} - \frac{24}{\frac{2}{5}} + 6$
 $= \frac{8}{5} - \frac{120}{2} + \frac{30}{5} = \frac{8}{5} - \frac{300}{5} + \frac{30}{5} = -\frac{262}{5}$
(c) $5b - 3c^2 = 5(\frac{2}{5}) - 3(-6)^2 = 2 - 108 = -106$

5. Evaluate the following expressions on your calculator:

Solution

(a)
$$(250/(34+56)) \times 27 = 75$$

(b) $23\frac{5}{7} + 6.3(4^5) = 6474.91$

(c)
$$3\sqrt{32} - \sqrt{15} = 13.0976$$

6. Simplify:

Solution

(a)
$$\frac{x^5 x^2}{x^{-3}} = x^{5+2-(-3)} = x^{10}$$

(b) $(x^{-2}y^3)^2 = x^{-2(2)}y^{3(2)} = x^{-4}y^6$
(c) $(x^{-5}y^4)^2(x^0y^{-2})^2 = x^{-10}y^4$

7. If there are .82 US dollars in one Camadian dollar, which is smaller, one US dollar, or one Canadian dollar?

Solution - One Canadian dollar.

8. One number is 6 times a second number. Find the numbers if their difference is 102.

Solution - Call the first number *x* and the second number *y*. Written algebraically, these two facts are:

$$x = 6y$$
$$x - y = 102$$

If we solve the second equation for x in terms of y, and then plug the equation for x into the first equation we get:

$$x = 102 + y$$
$$102 + y = 6y$$

Solving this equation for y we get:

$$102 = 5y \to \frac{102}{5} = y$$

Which we can then use to solve for *x*:

$$x = 102 + y \rightarrow x = 102 + \frac{102}{5} \rightarrow x = \frac{612}{5}$$

9. If you drive at an average speed of 65 miles per hour, how long will it take you to drive 530 miles? If you can bike a distance of 45 miles in three hours and 15 minutes, what is your average biking speed in miles per hour?

Solution

If you drive at 65 miles per hour it will take you:

$$\frac{530 miles}{65 \frac{miles}{hour}} \approx 8.15 hours$$

For the next part, we first note that three hours and 15 minutes is 3.25 hours, and so our average speed is:

$$\frac{45miles}{3.25hours} = 13.85 \frac{miles}{hour}$$

10. The length of a rectangle is 14 inches more than its width. If the area is 72 square inches, find the length and width of the rectangle.

Solution

We denote the length by l, and width by w, and the area by A. Put algebraically, these two requirements above are:

$$l = w + 14$$
$$A = lw = 72$$

If we plug the equation for l in the first line above into the equation for A in the second line above we get:

$$A = w(w + 14) = 72 \rightarrow w^2 + 14w = 72$$
$$\rightarrow w^2 + 14w - 72 = 0$$

To solve this for w we need to use the quadratic equation, with a = 1, b = 14 and c = -72.

$$w = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-14 \pm \sqrt{(-14)^2 - 4(1)(-72)}}{2(1)}$$
$$= \frac{-14 \pm 22}{2} = -7 \pm 11$$

We take the positive solution, as our width will have to be positive, and so we get w = 4 inches, which requires l = w + 14 = 18 inches.

11. Suppose that three-quarters of the freshmen live in a dorm. If tothirds of the freshmen dorm residents are women, what percentage of the freshman class are women who live in the dorm?

Solution

We want to calculate two-thirds of three-quarters. This is:

$$\left(\frac{3}{4}\right)\left(\frac{2}{3}\right) = \left(\frac{6}{12}\right) = \left(\frac{1}{2}\right) = .5 = 50\%$$

So, 50% of the freshman class are women who live in the dorm.

12. Solve for *x* in the following equations:

Solutions

(a) 3x - 5 = 9 + 7x

First we subtract 3x from both sides, giving us:

$$-5 = 9 + 4x$$

Then we subtract 9 from both sides, giving us:

$$-14 = 4x$$

Finally, to solve for x we divide both sides by 4 to get:

$$-\frac{14}{4}=x\rightarrow x=-\frac{7}{2}$$

(b) $x^2 - 5 = 31$

To solve this we first want to make one side 0, so we subtract 31 from both sides to get:

$$x^2 - 36 = 0$$

which we can then factor to get:

$$(x-6)(x+6) = 0$$

If the right hand side is 0 then one of the terms on the left must be 0, and so $x = \pm 6$.

(c)
$$x^2 - x - 12 = 0$$

To solve this one we factor the equation on the left hand side to get:

$$(x-4)(x+3) = 0$$

Again, if the left hand side is equal to 0, then one of the factors must be 0, and so the possible answers are 4, and -3.

(d)
$$\frac{x-3}{5} = \frac{x}{2}$$

To solve this we first multiply both sides by 5 to get:

$$x - 3 = \frac{5x}{2}$$

If we then multiply both sides by 2 we get:

2x - 6 = 5x

Subtracting 2x from both sides we get:

$$-6 = 3x$$

and if we then divide both sides by 3 we have our solution:

$$x = -2$$

(e) |x+3| = 10

If the absolute value is equal to 10, then the value is either 10 or -10. If the value is 10 then we have:

$$x + 3 = 10 \rightarrow x = 7$$

If on the other hand the value is equal to -10 we have:

$$x+3 = -10 \rightarrow x = -13$$

So, the two possible answers are 7 and -13.

13. Solve for *x* and *y*:

Solution - I'm not actually sure what this problem is asking, as there is no way to solve for both *x* and *y* given only one equation. So, I just simplified the equation into a line in slope-intercept form.

(a)
$$3x - 2y = 5$$

If we first add 2y to both sides, subtract 5 from both sides, and then divide both sides by 2 we get:

$$y = \frac{3}{2}x - \frac{5}{2}$$

(b) x + y = 7

If we subtract *x* from both sides we get:

$$y = -x + 7$$

14. Graph the line 5x - 2y = 6. What is the *y*-intercept?

Solution - I don't know how to draw a graph with the software I'm using to write these solutions, and I don't have time to learn right now. If you want to know how to graph this line, please ask me.

What we want to do is to figure out the equation for the line in slopeintercept form. To do this we add 2y to both sides, subtract 6 from both sides, and then divide both sides by 2 to get:

$$y = \frac{5}{2}x - 3$$

So, the slope of the line is $\frac{5}{2}$ and the y-intercept is -3.

15. A warehouse may contain bicycles, tricycles, and cars. Altogether there are 18 wheels in the warehouse. How many bicycles, tricycles, and cars are there? Give as many answers as possible.

Solution - This is a silly problem that just involves a bunch of guessing and checking. Don't worry about it. If you have any questions about it ask me, but I won't put anything like it on the quiz.

16. The palyground drawn below is in the shape of a rectangle with a semicircle attached as shown. Suppose that the longer side of the rectangle is twice the length of the shorter side and that the radius of the semicircle is 12 feet. What is the perimeter and the area of the playground?

Solution - Again, not sure how to draw with the program I'm using to make these solutions, so if you want to see the playground drawing please check the diagnostic test .pdf file posted on the website.

To solve this we first note that the width of the rectangle will be twice the radius of the semicircle, and so the width w is w = 24 feet. This implies that the length must be l = 48 feet. With this we can calculate both the perimeter:

$$P = 2l + w + \pi r = 2(48ft) + 24ft + \pi 12ft = 120ft + 12\pi ft$$

While the area will be:

$$A = wl + \frac{1}{2}\pi r^{2} = (24ft)(48ft) + \frac{1}{2}\pi (12ft)^{2} = 1152ft^{2} + 72\pi ft^{2}$$

17. Suppose that the ratio of undergraduate students to graduate students in an institution is 18:7. What percentage of the student body are graduate students?

Solution - The percentage will be:

$$\frac{7}{18+7} = .28 = 28\%$$

So, 28% of the student body are graduate students.

18. Suppose that your annual tuition as a freshman was \$1856. Each year tuition has increased 5%. Now you are in yoru senior year. What is your annual tuition?

Solution

When you reach your senior year it will be your third year of the 5% increase, and so your total tuition will be:

$$\$1856(1.05)^3 \approx \$2148$$

19. The company you work for was doing poorly two years ago and as a result everyone took at 10% cut in pay for the last year. The company is doing better now and the CEO has just promised to raise everyones salary 10% for next year. Does this mean that your salary next year will be the same as it was two years ago? Explain.

Solution - After the first year your salary will be .9 what it was when you started out. After the second year your salary will be $.9 \times 1.1 = .99$ what it was when you started out. So, it will be 1% less than your starting salary, not the same as your starting salary.

20. I'm not quite sure how to get the formatting right for this one. Don't worry about it. It won't be on the quiz.