Solutions

# Assignment 8

Math 1030

Due Monday, November 5th

1. **Logarithmic Scales and Inverse Square Laws**

   (a) How many times as much energy is released by an earthquake of magnitude 7 as by one of magnitude 5? (Problem 8D-20)

   A 1 magnitude difference is a difference of 32X in energy. So, a 2 magnitude difference is:

   $(32)^2 =$ **1,024 times as much energy**

   (b) What is the loudness, in decibels, of a sound 20 million times as loud as the softest audible sound? (Problem 8D-27)

   $I = 20{,}000{,}000 \times I_0$

   $I_0$ = softest audible sound

   $$\text{loudness in dB} = 10 \log_{10}\left(\frac{20{,}000{,}000\, I_0}{I_0}\right)$$

   $$\approx \boxed{73 \text{ dB}}$$

(c) How many times greater is the intensity of sound from a concert speaker at a distance of 1 meter than the intensity at a distance of 100 meters? (Problem 8D-32)

Intensity decreases with the square of the distance. So, a speaker 100 m away is $\frac{1}{(100)^2} = \frac{1}{10,000}$ times as loud as the speaker at 1 m. So, at 1 m the speaker is $\boxed{10,000}$ times as loud as at 100 m.

(d) What is the hydrogen (technically, hydronium) ion concentration of a solution with a pH of 2.5? (Problem 8D-38)

$$[H^+] = 10^{-pH} \text{ moles/L}$$

$$= \boxed{10^{-2.5} \text{ moles/L}}$$

$$= 3.16 \times 10^{-3} \text{ moles/L}$$

2. **Logarithms**

Solve each of the following equations for $x$:

(a) $15 = 10^x$

$\Rightarrow \log_{10} 15 = x \log_{10} 10$

$\Rightarrow x = \frac{\log_{10}(15)}{\log_{10}(10)} = \log_{10}(15) = \boxed{1.176}$

(b) $x = \log_5 7$

$x = \log_5 7 = \frac{\log_{10} 7}{\log_{10} 5} = \boxed{1.209}$

(c) $7 = 4 + 6^x$

$\Rightarrow 3 = 6^x$

$\Rightarrow \log_{10} 3 = x \log_{10} 6$

$\Rightarrow x = \frac{\log_{10} 3}{\log_{10}(6)} = \boxed{.613}$

(d) $\log_{10}(x) = 3.7$

$\Rightarrow 10^{\log_{10}(x)} = 10^{3.7}$

$\Rightarrow x = 10^{3.7} = \boxed{5,011.872}$

(e) $15,000 = 3 \times (4+3)^x$

$\Rightarrow 5,000 = (4+3)^x = 7^x$

$\Rightarrow \log_{10} 5,000 = x \log_{10} 7$

$\Rightarrow x = \frac{\log_{10} 5,000}{\log_{10} 7} = \boxed{4.377}$

(f) $18,000 = 2^{3^x}$

$\Rightarrow \log_{10}(18,000) = 3^x \log_{10}(2) \Rightarrow \frac{\log_{10}(18,000)}{\log_{10}(2)} = 3^x$

$\Rightarrow \log_{10}\left(\frac{\log_{10}(18,000)}{\log_{10}(2)}\right) = x \log_{10}(3) \Rightarrow x = \frac{\log_{10}\left(\frac{\log_{10}(18,000)}{\log_{10}(2)}\right)}{\log_{10}(3)}$

$\boxed{x = 2.911}$