Solutions

Assignment 7

Math 1030

Due Friday, October 26th

1. **Exponential and Linear Growth**

   Answer each of the following questions, and state whether it is an example of linear growth, exponential growth, or exponential decrease.

   (a) During the worst periods of hyperinflation in Brazil, the price of food increased at a rate of 30% per month. If your food bill was \$120 one month during this period, what was it three months later?

   $\$120\,(1.30)^3 = \boxed{\$263.64}$

   (b) The price of computer memory is decreasing at a rate of 12% per year. If a memory chip costs \$50 today, what will it cost in three years?

   $\$50\,(.88)^3 = \boxed{\$34.07}$

   (c) The population of Meadow View is increasing at a rate of 623 people per year. If the population is 2500 today, what will it be in four years?

   $2{,}500 + 623 \times 4 = \boxed{4{,}992 \text{ people}}$

2. **Doubling Time and Half-Life**

Answer each of the following questions concerning doubling times and half-lives.

(a) The doubling time of a bank account balance is 10 years. By what factor does it grow in 30 years? in 50 years?

In 30 years: $2^{(\frac{30}{10})} = 2^3 = \boxed{8}$

In 50 years: $2^{(\frac{50}{10})} = 2^5 = \boxed{32}$

(b) The initial population of a town is 15,600, and it grows with a doubling time of 8 years. What will the population be in 12 years? In 24 years?

In 12 years: $15,600 \times 2^{(\frac{12}{8})} = \boxed{44,123}$

In 24 years: $15,600 \times 2^{(\frac{24}{8})} = \boxed{124,800}$

(c) The half-life of a drug in the bloodstream is 4 hours. By what factor does the concentration of the drug decrease in 24 hours? In 36 hours?

In 24 hours: $(\frac{1}{2})^{(24/4)} = \boxed{\frac{1}{64} \approx 1.56\%}$

In 36 hours: $(\frac{1}{2})^{(36/4)} = \boxed{\frac{1}{512} \approx .195\%}$

(d) Radium-226 is a metal with a half-life of 1600 years. If you start with 1 kilogram of radium-226, how much will remain after 1000 years? after 10,000 years?

1000 yrs: $1\text{ kg} \times (\frac{1}{2})^{(1000/1600)} = \boxed{.64\text{ kg}}$

10,000 yrs: $1\text{ kg} \times (\frac{1}{2})^{(10,000/1600)} = \boxed{.013\text{ kg}}$

3. **Calculating Doubling Times and Half-Lives**

For each of the following use the approximate formula (rule of 70) to estimate either the doubling time or half life, depending on the problem, and then use the exact formula to calculate it precisely. Compare your answers.

(a) A city's population is growing at a rate of 3.5% per year. What is its doubling time?

Approximate $= \frac{70}{3.5} \approx \boxed{20 \text{ years}}$

Exact: $\frac{\log(2)}{\log(1.035)} = \boxed{20.15 \text{ years}}$

(b) Prices are rising at a rate of 0.9% per month. What is their doubling time? By what factor will prices increase in 1 year? In 8 years?

Approximate doubling time: $\frac{70}{.9} = \boxed{77.77 \text{ months}}$ ~~years~~

Exact doubling time: $\frac{\log(2)}{\log(1.009)} = \boxed{77.36 \text{ months}}$

Price increase in 1 year $= [(1.009)^{12} - 1] \times 100\%$
$= \boxed{11.35\%}$

Price increase in 8 years $= [(1.009)^{12 \times 8} - 1] \times 100\% = \boxed{136\%}$

(c) The production of a gold mine is declining by 6% per year. If its current annual production is 4000 kilograms, what will its production be in 15 years?

Approximate half-life: $\frac{70}{6} = \boxed{11.67 \text{ years}}$

Exact half-life: $\frac{-\log(2)}{\log(.94)} = \boxed{11.2 \text{ years}}$

Production in 15 years: $4000(.99)^{15}$
$= \boxed{1{,}581 \text{ kg/year}}$

4. **Logistic Population Growth**

Consider a population that begins growing exponentilly at a base rate of 4.0% per year and then follows a logistic growth pattern. If the carrying capacity is 60 million, find the actual growth rate when the population is 10 million, 30 million, and 50 million.

Growth rate at 10 million:

$$4.0\% \left(1 - \frac{10 \text{ million}}{60 \text{ million}}\right)$$

$$= \boxed{3.3\%}$$

Growth rate at 30 million

$$4.0\% \left(1 - \frac{30 \text{ million}}{60 \text{ million}}\right)$$

$$= \boxed{2.0\%}$$

Growth rate at 50 million

$$4.0\% \left(1 - \frac{50 \text{ million}}{60 \text{ million}}\right)$$

$$= \boxed{.7\%}$$