Assignment 6

Math 1030

Due Friday, October 19th

1. Compute the total cost *per year* of the first set of expenses. Then complete the sentence: On an *annual* basis, the first set of expenses is _% of the second set of expenses. (Taken from problems 26, 27, and 29 of section 4A from the textbook).

   (a) Marcus spends an average of \$4 a day on iTunes, his rent is \$350 per month.

   Annual iTunes spending $= (\$4/\text{day})(365\ \text{day}/\text{year}) = \boxed{\$1{,}460/\text{year}}$

   Annual rent $= (\$350/\text{month})(12\ \text{months}/\text{year}) = \boxed{\$4{,}200/\text{year}}$

   $\left(\frac{\$1{,}460/\text{year}}{\$4{,}200/\text{year}}\right) \times 100\% \approx \boxed{34.8\%}$

   (b) Sheryl buys a \$9 pack of cigarettes each week and spends \$30 a month on dry cleaning.

   Annual cigarette spending $= (\$9/\text{week})(52\ \text{weeks}/\text{year}) = \boxed{\$468/\text{year}}$

   Annual laundry spending $= (\$30/\text{month})(12\ \text{months}/\text{year}) = \boxed{\$360/\text{year}}$

   $\left(\frac{\$468/\text{year}}{\$360/\text{year}}\right) \times 100\% = \boxed{130\%}$

   (c) Vern drinks three 6-packs of beer each week at a cost of \$7 each and spends \$700 per year on his textbooks.

   Annual beer spending $= 3 \times (\$7/\text{week})(52\ \text{weeks}/\text{year}) = \boxed{\$1{,}092/\text{year}}$

   $\frac{\$1{,}092/\text{year}}{\$700/\text{year}} = \boxed{156\%}$

2. You currently drive 250 miles per week in a car that gest 21 miles per gallon of gas. You are considering buying a new fuel-efficient car for $16,000 (after trade-in on your current car) that gets 45 miles per gallon. Insurance premiums for the new and old car are $800 and $400 per year, respectively. You anticipate spending $1500 per year on repairs for the old car and having no repairs on the new car. Assume gas costs $3.50 per gallon. Over a five-year period, is it less expensive to keep your old car or buy the new car? (Problem 51 from section 4A of the textbook.)

Cost of old car:

$$(250\text{ miles/week})\left(\frac{1\text{ gallon}}{21\text{ miles}}\right)(\$3.50/\text{gallon})\left(\frac{52\text{ weeks}}{\text{year}}\right)(5\text{ years})$$
$$+ (\$800/\text{year})(5\text{ years}) + (\$1{,}500/\text{year})(5\text{ years}) = \boxed{\$22{,}333.30}$$

Cost of new car:

$$(250\text{ miles/week})\left(\frac{1\text{ gallon}}{45\text{ miles}}\right)(\$3.50/\text{gallon})\left(\frac{52\text{ weeks}}{\text{year}}\right)(5\text{ years})$$
$$+ (\$400/\text{year})(5\text{ years}) + \$16{,}000 = \boxed{\$23{,}055.60}$$

Barely less expensive to keep old car.

3. You could take a 15-week, three-credit college course, which requires 10 hours per week of your time and costs $500 per credit-hour of tuition. Or during those hours you could have a job paying $10 per hour. What is the net cost of the class compred to working? Given that the average college graduate earns nearly $20,000 per year more than a high school graduate, is it a worthwhile expense? (Problem 47 from section 4A of the textbook.)

Cost of course: $3 \times \$500 = \$1{,}500$

Money you could have made $= (\$10/\text{hour})\left(\frac{10\text{ hours}}{\text{week}}\right)(15\text{ weeks})$
$= \$1{,}500$

Total cost: $\$1{,}500 + \$1{,}500 = \boxed{\$3{,}000}$

If you make $20,000/year more it's definitely worth it.

4. Yancy invests $5000 in an account that earns simple interest at an annual rate of 5% per year. Samantha invests $5000 in a savings account with annual compounding at a rate of 5% per year. Make a table that shows the performance of both accounts for 5 years. The table should list the amount of interest earned each year and the balance in each account. Compare the balances after 5 years. (Problem 45 from section 4B of the textbook.)

	Simple Interest	Compound Interest
Year 1	Interest: $5,000 x 5% = $250 Total = $5,000+$250 = $5,250	Interest: $5,000 x 5% = $250 Total = $5,250
Year 2	Interest: $5,000 x 5% = $250 Total = $5,250 + $250 = $5,500	Interest: $5,250 x 5% = $262.50 Total = $5,250 + $262.50 = $5,512.50
Year 3	Interest = $250 total = $5,750	Interest: $5,521.50 x 5% = $275.63 Total = $5,512.50 + $275.63 = $5,788.13
Year 4	Interest = $250 Total = $6,000	Interest: $5,788.13 x 5% = $289.41 Total = $5,788.13 + $289.41 = $6,077.54
Year 5	Interest = $250 Total = $\boxed{\$6,250}$	Interest: $6,077.54 x 5% = $303.88 Total = $6,077.54 + $303.88 = $\boxed{\$6,381.42}$

Make $6381.42 − $6250
= $\boxed{\$131.42}$ more with compound interest

5. Use the compound interest formula to determine the accumulated balance after the stated period. Assume that interest is compounded annually. (Problems 47-52 from section 4B of the textbook.)

(a) $3000 is invested at an APR of 3% for 10 years.

$$A = P \times (1 + APR)^Y$$

$$A = \$3000 \times (1.03)^{10} = \boxed{\$4{,}031.75}$$

(b) $10,000 is invested at an APR of 5% for 20 years.

$$A = \$10{,}000 \times (1 + .05)^{20}$$

$$= \boxed{\$26{,}533}$$

(c) $40,000 is invested at an APR of 7% for 25 years.

$$A = \$40{,}000 \times (1 + .07)^{25}$$

$$= \boxed{\$217{,}097} \quad \text{(wow!)}$$

(d) $3000 is invested at an APR of 4% for 12 years.

$$A = \$3{,}000 \times (1.04)^{12}$$

$$= \boxed{\$4{,}803.10}$$

(e) $8000 is invested at an APR of 6% for 25 years.

$$A = \$8000 \times (1.06)^{25}$$

$$= \boxed{\$34{,}335.00}$$

(f) $40,000 is invested at an APR of 8.5% for 30 years.

$$A = \$40{,}000 \times (1.085)^{30}$$

$$= \boxed{\$462{,}330}$$

6. Use the compound interest formula for compounding more than once a year to determine the accumulated balance after the stated period. (Problems 53 through 60 from section 4B of the textbook.)

   (a) A $4000 deposit at an APR of 3.5% with monthly compounding for 10 years.

$$A = P\left(1+\frac{APR}{n}\right)^{(nY)}$$

$$= \$4000\left(1+\frac{.035}{12}\right)^{(12\times 10)} = \boxed{\$5,673.38}$$

   (b) A $2000 deposit at an APR of 3% with daily compounding for 5 years.

$$A = \$2000\left(1+\frac{.03}{365}\right)^{(365\times 5)}$$

$$= \boxed{\$2,323.65}$$

   (c) A $15,000 deposit at an APR of 5.6% with quarterly compounding for 20 years.

$$A = \$15,000\left(1+\frac{.056}{4}\right)^{(20\times 4)}$$

$$= \boxed{\$45,617.10}$$

   (d) A $10,000 deposit at an APR of 2.75% with montly compounding for 5 years.

$$A = \$10,000\left(1+\frac{.0275}{12}\right)^{(12\times 5)}$$

$$= \boxed{\$11,472.21}$$

(e) A \$2000 deposit at an APR of 7% with monthly compounding for 15 years.

$$A = \$2000\left(1+\frac{.07}{12}\right)^{(12\times 15)}$$

$$= \boxed{\$5,697.89}$$

(f) A \$3000 deposit at an APR of 5% with ~~adily~~ daily compounding for 10 years.

$$A = \$3000\left(1+\frac{.05}{365}\right)^{(365\times 10)}$$

$$= \cancel{\$4,946}.\ \boxed{\$4,945.99}$$

(g) A \$25,000 deposit at an APR of 6.2% with quarterly compounding for 30 years.

$$A = \$25,000\left(1+\frac{.062}{4}\right)^{(4\times 30)}$$

$$= \boxed{\$158,318.38}$$

(h) A \$15,000 deposit at an APR of 7.8% with monthly compounding for 15 years.

$$A = \$15,000\left(1+\frac{.078}{12}\right)^{(12\times 15)}$$

$$= \boxed{\$48,147.25}$$

7. Find the annual percentage yield (APY) for the banks described below. (Problems 61 through 64 from section 4B of the textbook.)

(a) A bank offers an APR of 3.5% compounded daily.

$$APY = \left[\left(1 + \frac{APR}{n}\right)^n - 1\right] \times 100\%$$
$$= \left[\left(1 + \frac{.035}{365}\right)^{365} - 1\right] \times 100\% = \boxed{3.56\%}$$

(b) A bank offers an APR of 4.5% compounded monthly.

$$APY = \left[\left(1 + \frac{.045}{12}\right)^{12} - 1\right] \times 100\%$$
$$= \boxed{4.59\%}$$

(c) A bank offers an APR of 4.25% compounded monthly.

$$APY = \left[\left(1 + \frac{.0425}{12}\right)^{12} - 1\right] \times 100\%$$
$$= \boxed{4.33\%}$$

(d) A bank offers an APR of 2.25% compounded quarterly.

$$APY = \left[\left(1 + \frac{.0225}{4}\right)^{4} - 1\right] \times 100\%$$
$$= \boxed{2.27\%}$$

8. Use the compound interest formula for continuous compounding to determine the accumulated balance after 1 year, 5 years, and 20 years. Also find the APY for each account. (Problems 65 through 67 of section 4B of the textbook.)

(a) A \$3000 desposit in an account with an APR of 4%.

$A = P \times e^{(APR \times Y)} \quad APY = [e^{(APR)} - 1] \times 100\%$

1 year

$A = \$3000 \times e^{(.04 \times 1)} = \boxed{\$3,122.43}$

5 years

$A = \$3000 \times e^{(.04 \times 5)} = \boxed{\$3,664.21}$

20 years

$A = \$3000 \times e^{(.04 \times 20)} = \boxed{\$6,676.62}$

$APY = (e^{.04} - 1) \times 100\% = \boxed{4.08\%}$

(b) A \$2000 deposit in an account with an APR of 5%.

$APY = (e^{APR} - 1) \times 100\%$

1 year

$A = \$2000 \times e^{(.05 \times 1)} = \boxed{\$2,102.54}$

5 years

$A = \$2000 \times e^{(.05 \times 5)} = \boxed{\$2,568.05}$

20 years

$A = \$2000 \times e^{(.05 \times 20)} = \boxed{\$5,436.56}$

$APY = (e^{.05} - 1) \times 100\% = \boxed{5.13\%}$

(c) A \$10,000 deposit in an account with an APR of 8%.

1 year

$A = \$10,000 \times e^{(.08 \times 1)} = \boxed{\$10,832.87}$

5 years

$A = \$10,000 \times e^{(.08 \times 5)} = \boxed{\$14,918.25}$

20 years

$A = \$10,000 \times e^{(.08 \times 20)} = \boxed{\$49,530.32}$

$APY = (e^{.08} - 1) \times 100\% = \boxed{8.33\%}$

9. At age 35 you start saving for retirement. If your investment plan pays an APR of 6% and you want to have $2 million when you retire in 30 years, how much should you deposit monthly? (Problem 54 from section 4C of the textbook.)

$$A = PMT \times \frac{\left[\left(1+\frac{APR}{n}\right)^{(nY)} - 1\right]}{\left(\frac{APR}{n}\right)}$$

So,

$$PMT = \frac{\$2,000,000}{\frac{\left[\left(1+\frac{.06}{12}\right)^{(12\times 30)} - 1\right]}{\left(\frac{.06}{12}\right)}} = \boxed{\$1,991.01/\text{month}}$$

10. Compute the total and annual return of the described investments. (Problems 59 through 61 of the textbook).

(a) Five years after buying 100 shares of XYZ stock you $60 per share, you sell the stock for $9400.

$$\text{Total Return} = \frac{\$9400 - \$60\times 100}{\$60\times 100} = \boxed{.5667}$$

$$\text{Annual Return} = \left(\frac{\$9400}{\$60\times 100}\right)^{(1/5)} - 1 = \boxed{.0939}$$

(b) You pay $8000 for a municipal bond. When it matures after 20 years, you receive $12,500.

$$\text{Total Return} = \frac{\$12,500 - \$8,000}{\$8,000} = \boxed{.5625}$$

$$\text{Annual Return} = \left(\frac{\$12,500}{\$8,000}\right)^{(1/20)} - 1 = \boxed{.0226}$$

(c) Twenty years after purchasing shares in a mutual fund for $6500, you sell them for $11,300.

$$\text{Total Return} = \frac{\$11,300 - \$6,500}{\$6,500} = \boxed{.7385}$$

$$\text{Annual Return} = \left(\frac{\$11,300}{\$6,500}\right)^{(1/20)} - 1 = \boxed{.0280}$$