Assignment 5

Math 1030

Due Friday, September 28th

Name: Solutions

1. **Percentage Calculations**

   (a) (Modification of question 6 from section 3A of the textbook.) Your receipt shows that you paid $47.96 for a new shirt, including sales tax. The sales tax rate is 9%. How much did you pay in sales tax?

   $\frac{\$47.96}{1.09} = \$44 = \text{price of shirt.}$

   $\$47.96 - \$44.00 = \boxed{\$3.96} = \text{tax}$

   (b) (Modification of question 8 from section 3A of the textbook.) A friend has a textbook that originally cost $150. The friend says that you can have it for 100% less than he paid for it. Your price will be:

   100% of $150 is $150. So, 100% less than he paid for it is:

   $\$150 - \$150 = \boxed{\$0}$

   So, it's free.

(c) (Question 97 from section 3A of the textbook.)
Your property taxes increase 6% in one year and increase 7% the next year. The total increase in taxes over the two years is 13%. Is this correct or incorrect? If incorrect, what is the actual percentage increase in the two years?

Incorrect. The actual increase is:

$100 \rightarrow 106$ 6% increase

$106 \rightarrow 113.42$ 7% increase

Total: $113.42 - 100 = 13.42\% \approx \boxed{13.4\%}$

(d) (Question 92 from section 3A of the textbook.)
Your dinner bill is $42.50. You leave $50. What percent tip did you leave?

$$\frac{\$50.00 - \$42.50}{\$42.50} \times 100\% \approx \boxed{17.6\%}$$

(e) (Question 81 from section 3A of the textbook.)
Express this change in terms of absolute change in terms of percentage points, and as a relative change in terms of percentage: Between 1960 and 2010, the percentage of the U.S. population composed of people over 65 increased from 9.2% to 13.2%.

Absolute Change: $13.2\% - 9.2\% = \boxed{4\%}$

Relative Change: $\frac{13.2\% - 9.2\%}{9.2\%} \times 100\% = \boxed{43.5\%}$

2. **Putting Numbers in Perspective**

(Question 82 from section 3B of the textbook.)

A few billion years from now, after exhausting its nuclear engines, the Sun will become a type of remnant star called a *white dwarf*. It will still have nearly the same mass (about $2 \times 10^{30} kg$) as the Sun today, but its radius will be only about that of the Earth (about 6500 km).

Note - Write all your answers in scientific notation.

(a) Calculate the average density of the white dwarf in units of kilograms per cubic centimeter.

$$\text{Density} = \frac{\text{mass}}{\text{volume}} = \frac{2\times10^{30}\,kg}{\frac{4}{3}\pi\,(6.5\times10^{8}\,cm)^3}$$

$$= 1.738\times10^{3}\,kg/cm^3 \approx \boxed{2\times10^{3}\,kg/cm^3}$$

(b) What is the mass of a teaspoon of material from the white dwarf? (Hint: A teaspoon is about 4 cubic centimeters.) Compare this mass to the mass of something familiar (for example: a person, a car, a tank.)

$$(2\times10^{3}\,kg/cm^3)(4\,cm^3) \approx \boxed{8\times10^{3}\,kg}$$

About the mass of a tank!

(c) A neutron star is a type of stellar remnant compressed to even greater densities that a white dwarf. Suppose that a neutron star has a mass 1.4 times the mass of the Sun but a radius of only 10 kilometers. What is its density? Compare the mass of 1 cubic centimeter of neutron star material to the total mass of Mt. Everest (about $5 \times 10^{10} kg$).

$$\text{Density} = \frac{(2 \times 10^{30} kg) \times 1.4}{\frac{4}{3} \pi (1 \times 10^{6} cm)^3}$$

$$= 6.68 \times 10^{11} kg/cm^3$$

$$\approx \boxed{7 \times 10^{11} kg/cm^3}$$

Over ten times the mass of Mt. Everest!

3. **Significant Figures and Rounding Rules**

Use the appropriate rounding rules to answer these questions with the correct precision or correct number of significant digits.

(a) (Question 70 from section 3C of the textbook).
Add the times 2 hours, 37 minutes and 1 hour, 22 minutes, 15 seconds.

2h 37min + 1h 22min 15sec
= 3h 59min

(b) (Question 72 from section 3C of the textbook).
Divide the distance 110 kilometers by the time 55 minutes.

$\frac{110\ km}{55\ min} = 2.0\ km/min$

(c) (Question 74 from section 3C of the textbook).
At the hardware store, you buy a 50-kilogram bag of sand. You also buy 1.25 kilograms of nails. What is the total weight of your purchases?

50 kg + 1.25 kg = 51.25 kg
the least ~~significant~~ precise significant digit is in the 10s place, so it's 50 kg

4. **Consumer Price Index**

(Question 46 from section 3D of the textbook.)

The typical (median) price of a single-family home in the West (United States) rose from \$129,600 in 1990 to \$241,300 in 2005. Calculate the relative change in price from 1990 to 2004, and compare it to the overall rate of inflation as measured by the Consumer Price Index.

Relative Rate: $\frac{\$241,300 - \$129,600}{\$129,600}$

$= .862 \times 100\% = \boxed{86.2\% \text{ increase}}$

Inflation:

$$\frac{CPI_{2004} - CPI_{1990}}{CPI_{1990}} \times 100$$

$$= \frac{188.9 - 130.7}{130.7} \times 100$$

$= 44.9 = 44.5\%$ increased.

So, housing increased much faster than inflation.