1. Find

(a) \[
\int_{0}^{\pi/2} \int_{0}^{5} x^2 \cos y \, dx \, dy
= \int_{0}^{\pi/2} \frac{x^3}{3} \cos y \bigg|_{0}^{5} \, dy
= \int_{0}^{\pi/2} \frac{125}{3} \cos y \, dy
= \frac{125}{3} \sin y \bigg|_{0}^{\pi/2}
= \frac{125}{3}
\]

(b) \[
\int_{0}^{1} \int_{0}^{1} \sqrt{x^2 + 5} \, dx \, dy
= \int_{0}^{1} \int_{0}^{x} \sqrt{x^2 + 5} \, dy \, dx
= \int_{0}^{1} \sqrt{x^2 + 5} y \bigg|_{0}^{x} \, dx
= \int_{0}^{1} \sqrt{x^2 + 5} \, dx
= \frac{1}{3}(x^2 + 5)^{3/2} \bigg|_{0}^{1}
= \frac{1}{3}(\sqrt{6} - \sqrt{5})
\]

2. Find the volume of the tetrahedron bounded by the coordinate planes and the plane \(2x + 3y + 4z - 12 = 0\)

\[
V = \int_{0}^{6} \int_{0}^{4-2x/3} \int_{0}^{3-x/2-3y/4} dz \, dy \, dx
= \int_{0}^{6} \int_{0}^{4-2x/3} (3-x/2-3y/4) \, dy \, dx
\]
\[
\int_0^6 (3y - xy/2 - 3y^2/8)\bigg|_0^{4-2x/3} dx \\
= \int_0^6 (12 - 2x - 2x + x^2/3 - 6 + 2x - x^2/6) dx \\
= \int_0^6 (6 - 2x + x^2/6) dx \\
= (6x - x^2 + x^3/18)|_0^6 \\
= 12
\]

3. Find the area of the part of the paraboloid \( z = x^2 + y^2 \) cut off by the plane \( z=9 \).

\[
A = \int \int_{x^2+y^2\leq 9} \sqrt{4x^2 + 4y^2 + 1} dxdy \\
= \int_0^{2\pi} \int_0^3 \sqrt{4r^2 + 1}rdrd\theta \\
= \int_0^{2\pi} \int_1^{\sqrt{37}} u^{1/2}u/8dud\theta \\
= \int_0^{2\pi} u^{3/2}/12|^{\sqrt{37}} d\theta \\
= \int_0^{2\pi} (37^{3/4} - 1)/12d\theta \\
= (37^{3/4} - 1)/6\pi
\]

4. Let \( \vec{F} = y\vec{i} + (x + z)\vec{j} + y\vec{k} \).

(a) Show that \( \vec{F} \) is conservative by computing \( \text{curl} \vec{F} \)

\[
M = y, \quad N = x + z, \quad P = y \\
M_y = 1 = N_x \\
N_z = 1 = P_y \\
P_x = 0 = M_z \\
\text{curl} \vec{F} = 0, \quad \text{domain of} \ \vec{F} \ \text{is the entire 3-space, hence} \ \vec{F} \ \text{is conservative}
\]

(b) Find the potential function \( f \).

\[
f_x = y \\
f = xy + g(y, z)
\]
\[f_y = x + z\]
\[f_y = x + g_y\]
\[g_y = z\]
\[g = yz + h(z)\]
\[f_z = y\]
\[f_z = y + h'\]
\[h' = 0\]
\[h = C\]
\[f = xy + yz + C\] (we can let \(C = 0\))

(c) Calculate \[\int_{(1,0,1)}^{(1,1,1)} \mathbf{F} \cdot d\bar{r}\]
\[
\int_{(1,0,1)}^{(1,1,1)} \mathbf{F} \cdot d\bar{r} = f|_{(1,0,1)}^{(1,1,1)} = 2
\]

5. (a) Use Green’s Theorem to calculate \[\int_C (e^{x^2} + y^2)dx + (x - \cos y^2)dy\], where \(C\) is positively oriented circle \((x - 2)^2 + y^2 = 16\)
\[
\int_C (e^{x^2} + y^2)dx + (x - \cos y^2)dy = \int \int_{(x-2)^2+y^2\leq 16} (1 - 2y)dxdy = 16\pi \text{ (integral of 1 gives the area of the disk, integral of } 2y \text{ is 0 due to symmetry)}
\]

(b) Calculate \[\int \int S \bar{F} \cdot \bar{n}dS\], where \(\bar{F} = \cos x \bar{i} + y^2 \bar{j} - z \bar{k}\) and \(S\) is the sphere \(x^2 + y^2 + z^2 = 1\)
\[
\int \int S \bar{F} \cdot \bar{n}dS = \int \int S (\sin x + 2y - 1)dV = -4\pi/3
\]
(integral of \(\sin x\) and \(2y\) are 0 due to symmetry, integral of 1 gives the volume)