1. (a) Change $(2, \pi/7, \pi/3)$ from spherical to cylindrical coordinates.
\[ \rho = 2 \]
\[ \theta = \pi/7 \]
\[ \phi = \pi/3 \]
The relationship between spherical and cylindrical coordinates is:
\[\theta\] is same, \[\rho = \rho \sin \phi,\] \[z = \rho \cos \phi.\]
Hence
\[ r = 2 \sin \pi/3 = \sqrt{3} \]
\[ \theta = \pi/7 \]
\[ z = 2 \cos \pi/3 = 1 \]
(b) Change \(x^2+y^2-2z^2=1\) from Cartesian to cylindrical coordinates.
\[ r = \sqrt{x^2+y^2} \text{ and } z = z \]
The equation is \(r^2 - 2z^2 = 1\)

2. Find
\[ \lim_{(x,y) \to (1,-1)} \frac{x^2}{x^2 + y^2} = \frac{1^2}{1^2 + (-1)^2} = \frac{1}{2} \]
\[ \lim_{(x,y) \to (0,0)} \frac{x^2}{x^2 + y^2} \]
- Along the line \((x,0)\), \[ \lim_{(x,y) \to (0,0)} \frac{x^2}{x^2 + y^2} = \lim_{(x,0) \to (0,0)} \frac{x^2}{x^2} = \lim_{x \to 0} 1 = 1 \]
- Along the line \((0,y)\), \[ \lim_{(x,y) \to (0,0)} \frac{x^2}{x^2 + y^2} = \lim_{(0,y) \to (0,0)} \frac{0}{y^2} = \lim_{0 \to 0} 0 = 0 \]
Hence the limit does not exist.
\[ \lim_{(x,y) \to (0,0)} \frac{xy}{\sqrt{x^2 + y^2}} \]
In polar coordinates, \(x = r \cos t, y = r \sin t\) and
3. Let \( f(x, y) = x^2 y - y^3 \). Find

(a) \( f_x = 2xy \)

(b) \( f_y = x^2 - 3y^2 \)

(c) \( \nabla f = <2xy, x^2 - 3y^2> \)

(d) \( D_{\vec{u}} f \) at \((1, 1)\), where \( \vec{u} = <0.6, 0.8> \)
\[
D_{\vec{u}} f(1, 1) = \nabla f(1, 1) \cdot \vec{u} = <2, -2> \cdot <0.6, 0.8> = -0.4
\]

(e) the direction of fastest increase at \((1, 1)\)
\[
<2, -2>
\]

(f) the equation of the tangent plane at \((1, 1)\)
\[
z = f(1, 1) + \nabla f(1, 1) \cdot <x - 1, y - 1>
\]
\[
z = 2(x - 1) - 2(y - 1)
\]

4. Find the global minimum and the global maximum of \( f(x, y) = 3x + 2y \) on \( S \) if

(a) \( S = \{(x, y) : x^2 + \frac{1}{4}y^2 \leq 1\} \)
\( \nabla f = <3, 2> \), hence there are no stationary points, and the maximum occurs on the boundary. Parametrize boundary:
\[
x = \cos t, y = 2 \sin t, \ t \in [0, 2\pi]
\]
\[
f|_{\partial S} = 3 \cos t + 4 \sin t
\]
\[
\frac{df}{dt} = -3 \sin t + 4 \cos t
\]
\[
\frac{df}{dt} = 0 \text{ when } \tan t = \frac{4}{3}
\]
There are two such points: \((0.6, 1.6)\) and \((-0.6, -1.6)\)
\[
f(0.6, 1.6) = 5 \text{ and } f(-0.6, -1.6) = -5
\]
5 is max, -5 is min.

(b) \( S = \{(x, y) : x^2 - y^2 = \frac{5}{4}\} \)
There is no min or max (set \( S \) is not bounded).

5. The pressure, temperature and volume of a certain gas are related by \( PV = 5T \). The pressure is increasing at a rate of 2 pounds per square inch per second, and the volume is decreasing at a rate of 3 cubic
inches per second. At what rate is the temperature changing when the pressure is 10 pounds per square inch and the volume is 20 cubic inches?

The temperature $T$ is a function of pressure $P$ and volume $V$, which are both functions of time $t$. Therefore, the temperature is a function of $t$ as well, and to find its rate of change with respect to time, we need the chain rule.

$$\frac{dT}{dt} = \frac{\partial T}{\partial P} \cdot \frac{dP}{dt} + \frac{\partial T}{\partial V} \cdot \frac{dV}{dt}$$

$$\frac{\partial T}{\partial P} = \frac{V}{5}$$
$$\frac{\partial T}{\partial V} = \frac{P}{5}$$
$$\frac{dP}{dt} = 2$$
$$\frac{dV}{dt} = -3$$
$$\frac{dT}{dt} = 2\frac{V}{5} - 3\frac{P}{5}$$
$$\frac{dT}{dt}(10, 20) = 2$$