1. Find

(a) \( \lim_{x \to 0} \frac{\sin x}{\tan (3x)} \)
\[= \lim_{x \to 0} \frac{\sin x}{x} \cdot \frac{3x}{\sin (3x)} \cdot \frac{\cos (3x)}{3} \]
\[= \frac{1}{3} \]

(b) \( \lim_{x \to 1} \frac{x^2 + x - 2}{x^2 - 1} \)
\[= \lim_{x \to 1} \frac{(x + 2)(x - 1)}{(x - 1)(x + 1)} \]
\[= \lim_{x \to 1} \frac{x + 2}{x + 1} \]
\[= \frac{3}{2} \]

(c) \( \lim_{x \to 0^+} \frac{|x|}{x} \)
\[= \lim_{x \to 0^+} \frac{x}{x} \]
\[= 1 \]

2. Find \( \frac{dy}{dx} \) if \( x \cos (xy) + y = 1 \).
\[\cos (xy) - x \sin (xy)(y + y') + y' = 0 \]
\[y'(1 - x^2 \sin (xy)) = xy \sin (xy) - \cos (xy) \]
\[y' = \frac{xy \sin (xy) - \cos (xy)}{1 - x^2 \sin (xy)} \]

3. Find \( y' \)
(a) \[ y = \frac{2x - 3}{2x + 3} \]
\[ y' = \frac{2(2x + 3) - 2(2x - 3)}{(2x + 3)^2} \]
\[ y' = \frac{12}{(2x + 3)^2} \]

(b) \[ y = \cos^2 t - \cos^2 t \]
\[ y' = 2t \sin^2 t + 2 \cos t \sin t \]

(c) \[ y = t^3 \sin t \]
\[ y' = 3t^2 \sin t + t^3 \cos t \]

4. Determine the intervals where \( y = x^3 - \frac{6}{5}x^5 \) is decreasing and where it is concave down.
\[ y' = 3x^2 - 6x^4 = 3x^2(1 - 2x^2) \]
\[ y'' = 6x - 24x^3 = 6x(1 - 4x^2) \]
\[ y \text{ is decreasing when } y' \text{ is negative, which is on } (-\infty, -1/\sqrt{2}) \text{ and } (1/\sqrt{2}, \infty) \]
\[ y \text{ is concave down when } y'' \text{ is negative, which is on } (-1/2, 0) \text{ and } (1/2, \infty) \]

5. The right circular cylinder is to have a volume \( 250\pi \) cube inches. Determine its radius \( r \) and height \( h \) so that its area is minimal. The volume of a cylinder is \( V = \pi r^2 h \), the area is \( A = 2\pi(r + h)r \).
\[ \pi r^2 h = 250\pi \]
\[ h = 250r^{-2} \]
\[ A = 2\pi(r + 250r^{-2})r = 2\pi(r^2 + 250r^{-1}) \]
\[ A' = 2\pi(2r - 250r^{-2}) \]
\[ A' = 0 \text{ when } r = 5, \text{ check this is minimum.} \]
\[ h = 250/25 = 10 \]

6. Find
(a) \[ \int \frac{y^5 - 1}{(y^6 - 6y + 1)^2} \, dy \]
\[ = \int u = y^6 - 6y + 1, du = (6y^5 - 6) \, dy \]
\[
\int \frac{1}{6}u^{-2}du = -u^{-1/6} + C = -\frac{1}{6(y^6 - 6y + 1)} + C
\]

(b) \[
\int_0^{\pi/2} \sqrt{\sin t \cos t} dt = \left| u = \sin t, du = \cos t dt \right|
\]
\[
= \int_0^1 u^{1/2} du
\]
\[
= \frac{2}{3}u^{3/2} \bigg|_0^1
\]
\[
= \frac{2}{3}
\]

7. Find the general solution of the differential equation \( y' = t^3 y^6 \).
\[
y^{-6} dy = t^3 dt
\]
\[
\int y^{-6} dy = \int t^3 dt
\]
\[
y^{-5} / 5 = t^4 / 4 + C
\]
\[
y^{-5} = -5/4t^4 + C
\]
\[
y = (-5/4t^4 + C)^{-1/5}
\]
\[
y = \frac{1}{\sqrt[5]{-5/4t^4 + C}}
\]

8. A ball is thrown directly upward from the 32 feet high roof with an initial velocity of 16 feet per second. In how many seconds will it strike the ground and with what velocity? The gravity is \( g = 32 \text{ ft/s}^2 \).
\[
s'' = -32
\]
\[
s' = -32t + 16
\]
\[
s = -16t^2 + 16t + 32
\]
\[
s = 0 \text{ when } t^2 - t - 2 = 0
\]
\[
t = -1 (impossible) \text{ and } t = 2
\]
\[
s'(2) = -48 (48 \text{ feet per second, going down})
\]

9. Find the volume of the solid obtained by rotating the region bounded by \( y = x^2 - 2x + 1 \) and \( y = 1 \) about the y-axis.

intersection points:
\[ x^2 - 2x = 0 \]
\[ x = 0 \text{ and } x = 2 \]
\[ V = \int_0^2 2\pi x(2x - x^2)\,dx \]
\[ V = 2\pi \int_0^2 (2x^2 - x^3)\,dx \]
\[ V = 2\pi (2x^3/3 - x^4/4)|_0^2 \]
\[ V = 2\pi (16/3 - 4) = 8/3\pi \]

10. An upright cylindrical tank has radius 2 feet and is 5 feet deep. If the water in the tank is 4 feet deep, how much work is done in pumping all the water over the top edge of the tank? The density of water is 62.4 pounds per cubic foot.

Let \( h \) denote the height of the water level.
\[ \Delta W = 4\pi(5 - h)62.4\Delta h \]
\[ W = 62.4\pi \int_0^4 (20 - 4h)\,dh \]
\[ W = 62.4\pi (20h - 2h^2)|_0^4 \]
\[ W = 48 \cdot 62.4\pi \]