1. Find $y'$ of

(a) $y = x^5 \sin x$
$$y' = 5x^4 \sin x + x^5 \cos x$$

(b) $y = \frac{x + 2}{x - 2}$
$$y' = \frac{x - 2 - (x + 2)}{(x - 2)^2} = \frac{-4}{(x - 2)^2}$$

(c) $y = \sqrt{3x - 2} = (3x - 2)^{1/2}$
$$y' = \frac{1}{2} \cdot (3x - 2)^{-1/2} \cdot 3 = \frac{3}{2\sqrt{3x - 2}}$$

(d) $y = (x^3 - 2x + 1)^9$
$$y' = 9(x^3 - 2x + 1)^8(3x^2 - 1)$$

2. (a) Find $\frac{dy}{dx}$ if $y^4 - y = x^5$

Differentiate both sides with respect to $x$:
$$4y^3 \frac{dy}{dx} - \frac{dy}{dx} = 5x^4$$
$$\frac{dy}{dx}(4y^3 - 1) = 5x^4$$
$$\frac{dy}{dx} = \frac{5x^4}{4y^3 - 1}$$

(b) Find $y^{(10)}$ if $y = 543x^9 - 98x^7$
$$y^{(10)} = 0$$ since degree of $y$ is less than 10.

3. A farmer wants to fence off a rectangular pen. The fence for two parallel side costs $10 per foot, and the fence for the other two sides costs $15 per foot. He can spend $600. What are the dimensions of the largest pen he can enclose?
Let $x$ denote the length of sides that are $10$ per foot, and $y$ the length of sides that are $15$ per foot. Then the price of the fence is

$$20x + 30y = 600$$

Solve for $x$ to get

$$x = 30 - 1.5y$$

The area of the pen is

$$A = xy$$

Substituting $30 - 1.5y$ for $x$, get

$$A(y) = 30y - 1.5y^2$$

We need to maximize the area.

$A'(y) = 30 - 3y$

$A'(y) = 0$ if $y = 10$

Check that this gives maximum.

Then $x = 30 - 1.5 \cdot 10 = 15$.

The maximum area is $15 \cdot 10 = 150$ square feet.

4. A plane flying at a constant altitude of 3 miles at a constant speed of 1 mile per second passes directly over an observer. How fast is the distance from the observer increasing 4 seconds later? (You may need Pythagorean theorem: $a^2 + b^2 = c^2$, $c$ is the hypotenuse.)

Let $x$ denote the distance from the observer, and $y$ the horizontal distance. Then

$$x^2 = 3^2 + y^2$$

4 seconds later, $y = 4$, and $x^2 = 3^2 + 4^2 = 25$, hence $x = 5$.

Differentiating both sides with respect to time, get:

$$2x \frac{dx}{dt} = 2y \frac{dy}{dt}$$

$$\frac{dy}{dt} = 1$$

$$\frac{dx}{dt} = 4 \cdot 1/5 = 0.8$$
5. Let \( f(x) = x^3 + 3x^2 - 24x + 10 \)

(a) Determine where is \( f \) increasing/decreasing.
\[
f'(x) = 3x^2 + 6x - 24 = 3(x^2 + 2x - 8) = 3(x + 4)(x - 2)
\]
\( f'(x) = 0 \) if \( x = -4 \) or \( x = 2 \)
Test a point in each interval.
On \((-\infty, -4)\) and \((2, \infty)\) \( f' \) is positive and \( f \) is increasing. On \((-4, 2)\) \( f' \) is negative and \( f \) is decreasing.

(b) Determine where is \( f \) concave up/down.
\[
f''(x) = 6x + 6
\]
\( f''(x) = 0 \) when \( x = -1 \)
Test a point in each interval.
On \((-\infty, -1)\) \( f'' \) is negative, and \( f \) is concave down. On \((-1, \infty)\) \( f'' \) is positive and \( f \) is concave up.

(c) Use your answers to sketch the graph of \( f \).

6. Function \( y = \frac{x^2 - 4}{x - 2} \) is not defined at a certain point. How should it be defined at that point in order to be continuous?

\( y \) is not defined at \( x = 2 \).
\[
\lim_{x \to 2} y = \lim_{x \to 2} \frac{x^2 - 4}{x - 2} = \lim_{x \to 2} (x + 2) = 4
\]
Therefore \( f(2) \) should be 4.

7. Show that equation \( x^7 + x^3 + x + 2 = 0 \) has a solution on \([-1, 1]\).

Let \( f(x) = x^7 + x^3 + x + 2 \)
The solution \( c \) to the given equation satisfies \( f(c) = 0 \)
\( f(-1) = -1 \) and \( f(1) = 5 \).
Since \( f \) is continuous on \([-1, 1]\), and 0 is between -1 and 5, IVT implies there exists \( c \) between -1 and 1 so that \( f(c) = 0 \).