

MATH 1010-4

FINAL EXAM SOLUTIONS

December 13, 2005

- (Fractions) Simplify $\frac{2}{3} - \frac{3}{4} \cdot \frac{2}{5} = \frac{2}{3} - \frac{3 \cdot 2}{4 \cdot 5} = \frac{2}{3} - \frac{3}{10} = \frac{20}{30} - \frac{9}{30} = \frac{11}{30}$
- (A Linear Equation) Solve the equation $5(1 - x) + 2 = 1 - 3(2x + 1)$.
$$5 - 5x + 2 = 1 - 6x - 3$$
$$7 - 5x = -6x - 2$$
$$6x - 5x = -2 - 7$$
$$x = -9$$
- (A Quadratic Equation) Solve the equation $x^2 - 10x + 15 = 0$.
$$x^2 - 2 \cdot 5x = -15$$
$$x^2 - 2 \cdot 5x + 25 = -15 + 25$$
$$(x - 5)^2 = 10$$
$$x - 5 = \sqrt{10} \text{ or } x - 5 = -\sqrt{10}$$
$$x = 5 + \sqrt{10} \text{ or } x = 5 - \sqrt{10}$$
- (A Rational Equation) Solve the equation $\frac{4}{x-3} - \frac{6}{x+1} - 1 = 0$
$$(x-3)(x+1) \cdot \frac{4}{x-3} - (x-3)(x+1) \cdot \frac{6}{x+1} - (x-3)(x+1) \cdot 1 = 0$$
$$4(x+1) - 6(x-3) - (x^2 - 3x + x - 3) = 0$$
$$4x + 4 - 6x + 18 - (x^2 - 2x - 3) = 0$$
$$-2x + 22 - x^2 + 2x + 3 = 0$$
$$-x^2 + 25 = 0$$
$$x^2 = 25$$
$$x = 5 \text{ or } x = -5$$

5. (A Radical Equation) Solve the equation $\sqrt{2x+5} + 2 = 5$.

$$\sqrt{2x+5} = 3$$

$$2x + 5 = 9$$

$$2x = 4$$

$$x = 2$$

For the next three question let

$$f(x) = \frac{x+3}{x^2-5x}$$

6. (Domain) What is the natural domain of f ?

$$x^2 - 5x \neq 0$$

$$x(x-5) \neq 0$$

$$x \neq 0 \text{ and } x \neq 5$$

7. (Evaluating at a number) Find $f(4)$,

$$f(4) = \frac{4+3}{4^2-5 \cdot 4} = \frac{7}{16-20} = \frac{7}{-4} = -\frac{7}{4}$$

8. (Evaluating at an expression) Find $f(x+2)$.

$$f(x+2) = \frac{(x+2)+3}{(x+2)^2-5 \cdot (x+2)} = \frac{x+5}{x^2+4x+4-5x-10} = \frac{x+5}{x^2-x-6}$$

9. (Polynomials) Write the following polynomial expression in standard form. What is its degree and its leading coefficient?

$$(x^3-1)(x+5) - (2x^3+2) = x^4+5x^3-x-5-2x^3-2 = x^4+3x^3-x-7$$

The degree is 4, the leading coefficient is 1.

10. (Complex numbers) Calculate $\frac{1-2i}{2+3i}$. Your answer should be in the form $a+bi$ where a and b are real numbers.

$$\frac{1-2i}{2+3i} = \frac{(1-2i)(2-3i)}{(2+3i)(2-3i)} = \frac{2-3i-4i-6}{4+9} = \frac{-4-7i}{13} = -\frac{4}{13} - \frac{7}{13}i$$

11. (Linear System) Solve the system

$$2x - y = 5$$

$$5x + 2y = 17$$

$$4x - 2y = 10$$

$$9x = 27$$

$$x = 3$$

$$2 \cdot 3 - y = 5$$

$$y = 6 - 5$$

$$y = 1$$

12. (Radical Expressions) Simplify the expression

$$\frac{\sqrt{x^3}}{\sqrt[3]{x^4}} = \frac{x^{3/2}}{x^{4/3}} = x^{3/2-4/3} = x^{9/6-8/6} = x^{1/6}$$

13. (Straight Lines) Find an equation of the line that passes through (3, 5) and has slope $-\frac{1}{3}$. Draw its graph.

$$y - 5 = -\frac{1}{3}(x - 3)$$

$$y = -\frac{1}{3}x + 1 + 5$$

$$y = -\frac{1}{3}x + 6$$

To draw a graph, find another point on the line. For instance, if $x = 0$, $y = 6$.

The graph is a line passing through (3, 5) and (0, 6)

14. (Distance) Find the distance between the points (3, 2) and (5, -1).

$$d = \sqrt{(-1 - 2)^2 + (5 - 3)^2} = \sqrt{3^2 + 2^2} = \sqrt{13}$$

15. (Powers) Simplify the expression (i.e. write with only positive exponents, so that a , b and c occur only once)

$$\frac{(a^2b^{-1}c^{-2})^3}{(ab^2c^3)^{-2}} = \frac{a^6b^{-3}c^{-6}}{a^{-2}b^{-4}c^{-6}} = a^{6-(-2)}b^{-3-(-4)}c^{-6-(-6)} = a^8b$$

16. (Rational Expressions) Simplify the expression

$$\frac{1}{x-2} - \frac{3}{x+3}$$

$$\begin{aligned} LCD &= (x-2)(x+3) \\ \frac{1}{x-2} - \frac{3}{x+3} &= \frac{x+3}{(x-2)(x+3)} - \frac{3(x-2)}{(x-2)(x+3)} \\ &= \frac{x+3-3(x-2)}{(x-2)(x+3)} \\ &= \frac{x+3-3x+6}{(x-2)(x+3)} \\ &= \frac{-2x+9}{(x-2)(x+3)} \end{aligned}$$

17. (Rational Expressions) Simplify the expression

$$\begin{aligned} \frac{x^2-9}{x^2-6x+9} \div \frac{x}{x^2-3x} &= \frac{x^2-9}{x^2-6x+9} \cdot \frac{x^2-3x}{x} \\ &= \frac{(x-3)(x+3)}{(x-3)^2} \cdot \frac{x(x-3)}{x} \\ &= \frac{(x+3)}{(x-3)} \cdot \frac{(x-3)}{1} \\ &= x+3 \end{aligned}$$

18. (A Word Problem) The length of the playground is 20 feet greater than the width. Its area is 2,400 square feet. Find its dimensions.

Let l denote the length, and w denote the width. Then

$$l = w + 20$$

$$l \cdot w = 2400$$

$$(w + 20)w = 2400$$

$$w^2 + 20w = 2400$$

$$w^2 + 2 \cdot 10w + 100 = 2400 + 100$$

$$(w + 10)^2 = 2500$$

$$w + 10 = 50 \text{ or } w + 10 = -50$$

$$w = 40 \text{ or } w = -60$$

Since width cannot be negative, $w = 40$.

$$l = 40 + 20$$

$$l = 60$$

19. (A Word Problem) Working together, Erin and Bobby take 12 hours to paint their apartment. Working alone, Bobby takes 10 hours longer than Erin. How long would it take each of them to paint it alone?

Let t denote the time it takes Erin to paint the apartment. Then the time it takes Bobby is $t + 10$. In one hour Erin paints $\frac{1}{t}$ of the apartment, while Bobby paints $\frac{1}{t+10}$. Together they paint $\frac{1}{t} + \frac{1}{t+10}$ of the apartment. We also know they need 12 hours painting together, hence in an hour they'll paint $\frac{1}{12}$ of the apartment. Therefore

$$\frac{1}{t} + \frac{1}{t+10} = \frac{1}{12}$$

$$LCD = 12t(t + 10)$$

$$12t(t + 10)\frac{1}{t} + 12t(t + 10)\frac{1}{t+10} = 12t(t + 10)\frac{1}{12}$$

$$12(t + 10) + 12t = t(t + 10)$$

$$24t + 120 = t^2 + 10t$$

$$t^2 - 14t = 120$$

$$t^2 - 2 \cdot 7t + 49 = 120 + 49$$

$$(t - 7)^2 = 169$$

$$t - 7 = 13 \text{ or } t - 7 = -13$$

$$t = 20 \text{ or } t = -6$$

Time cannot be negative, therefore it takes Erin 20 hours to paint the apartment alone. Bobby takes 10 hours more, thus 30 hours.

20. (A Word Problem) A company mixes blueberries and strawberries in a 'Frozen Berry Mix'. Blueberries cost \$9 per pound, and strawberries cost \$5 per pound. How many of each should be mixed to obtain a 16oz (1 pound) package worth \$6?

Let b denote the amount of blueberries in pounds, and s the amount of strawberries. Then the cost of the mix is

$$9b + 5s = 6$$

we also know that

$$b + s = 1$$

Solve this system:

$$-5b - 5s = -5$$

$$4b = 1$$

$$b = \frac{1}{4}$$

$$s = 1 - b$$

$$s = 1 - \frac{1}{4}$$

$$s = \frac{3}{4}$$

Therefore, there should be a quarter pound (4oz) of blueberries, and three quarter pounds (12oz) of strawberries in the mix.