Math 2270-3
Spring 2005
January 25, 2005

Name: KEY

Quiz I

Instructions: This quiz is a total of 30 points with each question worth 10 points. Answer each question carefully and thoughtfully to receive full credit. Partial credit will be awarded, and points will be deducted if you write the answer down to a problem without justifying your steps. You do not need to simplify your answer unless it helps for clarity. Calculators are not permitted (nor are needed) for this quiz.

1. Solve the following system of equations using Gauss-Jordan elimination, showing all steps:

$$\begin{aligned} x + y + 2z &= 1 \\ 2x + 3y - 2z &= 2 \\ -x + y + 6z &= 39 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 2 & 3 & -2 & 2 \\ -1 & 1 & 6 & 39 \end{array}\right] \xrightarrow[\mathrm{III}+\mathrm{I}]{\mathrm{II}-2(\mathrm{I})} \left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & 1 & -6 & 0 \\ 0 & 2 & 8 & 40 \end{array}\right] \xrightarrow{\mathrm{III}-2(\mathrm{II})} \left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & 1 & -6 & 0 \\ 0 & 0 & 20 & 40 \end{array}\right] \xrightarrow{\frac{1}{20}\mathrm{III}} \left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & 1 & -6 & 0 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$$\xrightarrow[\mathrm{I}-2(\mathrm{III})]{\mathrm{II}+6(\mathrm{III})} \left[\begin{array}{ccc|c} 1 & 1 & 0 & -3 \\ 0 & 1 & 0 & 12 \\ 0 & 0 & 1 & 2 \end{array}\right] \xrightarrow{\mathrm{I}-\mathrm{II}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -15 \\ 0 & 1 & 0 & 12 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$$\boxed{\begin{aligned} x &= -15 \\ y &= 12 \\ z &= 2 \end{aligned}}$$

2. For the following three systems, determine whether or not they have a unique solution, no solution, or infinitely many solution and justify your answer. *Note: You should be able to answer this* ***by inspection****. Do not use Gauss-Jordan elimination.*

a. The augmented matrix

$$\left[\begin{array}{cccc|c} 1 & 4 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 0 & 1 \end{array}\right] \rightarrow \text{inconsistent}$$

because the last row is 0 augmented to a constant

b. The augmented matrix

$$\left[\begin{array}{cccccc|c} 1 & 0 & 0 & 4 & 5 & 3 & 1 \\ 0 & 1 & 0 & 0 & -1 & 0 & 2 \\ 0 & 0 & 1 & 3 & 0 & 0 & 3 \\ 0 & 0 & 0 & 1 & 0 & -20 & 4 \\ 0 & 0 & 0 & 0 & 1 & 3 & -5 \\ 0 & 0 & 0 & 0 & 0 & 1 & 6 \end{array}\right]$$

unique $\rightarrow$ all rows have leading ones

c. The system of equations:

$$\begin{aligned} x + 2y + 3z &= 5 \\ 4x + 8y + 12z &= 20 \\ -10x - 20y + z &= -30 \end{aligned}$$

infinitely many: $II = 4(I)$ and III is not a multiple of row I, so consistent

3. Suppose $\vec{v}_1, \vec{v}_2, ..., \vec{v}_m$ are arbitrary vectors in $\mathbb{R}^n$. Consider the transformation from $\mathbb{R}^m$ to $\mathbb{R}^n$:

$$T\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{bmatrix} = x_1\vec{v}_m + x_2\vec{v}_{m-1} + \cdots + x_m\vec{v}_1$$

$$= \sum_{i=1}^{m} x_i \vec{v}_{m-i+1}$$

a. Prove that T is a linear transformation.

b. Find the matrix A such that $T(\vec{x}) = A\vec{x}$.

a. Show $T(\vec{x}+\vec{y}) = T(\vec{x}) + T(\vec{y})$

$$\sum_{i=1}^{m} (x_i + y_i)\vec{v}_{m-i+1} = \sum_{i=1}^{m} x_i \vec{v}_{m-i+1} + y_i \vec{v}_{m-i+1} = \sum x_i \vec{v}_{m-i+1} + \sum y_i \vec{v}_{m-i+1}$$

$$= T(\vec{x}) + T(\vec{y})$$

& $T(k\vec{x}) = k\,T(\vec{x})$

$$\sum (kx_i)\vec{v}_{m-i+1} = \sum k x_i \vec{v}_{m-i+1} = k \sum_{i=1}^{m} x_i \vec{v}_{m-i+1} = k\,T(\vec{x})$$

b. $A = \begin{bmatrix} \vec{v}_m & \vec{v}_{m-1} & \cdots & \vec{v}_1 \end{bmatrix}$ (follows from definition of matrix vector mult)