

Practice Final Exam

Instructions: This is a practice final. The length is what I would expect of you in the exam, however I will not limit myself to these questions. It is recommended that you review your old quizzes and exams as well as interesting homework questions as well.

1. Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & -4 \\ 0 & 0 & 4 \end{bmatrix}$$

- Diagonalize A .
 - Find an expression for A^t .
 - What are the eigenvalues and eigenvectors of A^3 ? Can you relate these to the eigenvalues and eigenvectors of A ?
 - Generalize what you found in part c.
2. If A is a 2 by 2 matrix with $\text{tr}(A) = 0$ and $\det A = 9$, what are the eigenvalues of A ?
3. Find the 3-volume of the 3-parallelepiped defined by the vectors:

$$\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

4. This question asks about the following matrix:

$$A = \begin{bmatrix} 4 & 3 \\ 4 & 8 \end{bmatrix}$$

- What are the eigenvalues of A ?
 - If A is the matrix belonging to the discrete dynamical system $\vec{x}_{t+1} = A\vec{x}_t$, is 0 a stable or unstable equilibrium point? Why or why not?
 - If A is the matrix belonging to the continuous differential equation $\frac{d\vec{x}}{dt} = A\vec{x}$, is 0 a stable or unstable equilibrium point? Why or why not?
5. Find an orthonormal eigenbasis for the following matrix:

$$A = \begin{bmatrix} 0 & 2 & 2 \\ 2 & 1 & 0 \\ 2 & 0 & -1 \end{bmatrix}$$

6. Consider the following transformation from $\mathbb{R}^{2 \times 2} \rightarrow \mathbb{R}^{2 \times 2}$:

$$T(M) = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} M,$$

with respect to the following basis \mathcal{B} :

$$\mathcal{B} = \left(\begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & -1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 2 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 2 \end{bmatrix} \right)$$

- a. Find the A matrix of this transformation.
- b. Find bases for the image and the kernel of A .
- c. Find bases for the image and the kernel of T .
- d. Determine the rank and the nullity of the transformation T .

7. Find bases for the image and the kernel of:

$$A = \begin{bmatrix} 1 & 2 & 2 & -5 & 6 \\ -1 & -2 & -1 & 1 & 1 \\ 4 & 8 & 5 & -8 & 9 \\ 3 & 6 & 1 & 5 & -7 \end{bmatrix}$$

8. Find the QR factorization of the matrix

$$M = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 2 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}.$$

9. Prove that if A is not an invertible matrix, then $\lambda = 0$ is an eigenvalue of A .

10. Find a solution of the system:

$$\begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix} x = \begin{bmatrix} 0 \\ 3 \end{bmatrix}.$$

If no such solution can be found, then find a least squares solution.