5.5: 3, 15, 16

8.1 10, 14, 42

9.1 31, 52

9.3 7

Genetics

5.5

3. $\langle x,y\rangle = (Sx)^T Sy = Sx \cdot Sy$

a. $\langle x,y\rangle = Sy \cdot Sx = \langle y, x\rangle$

b. $\langle x+y, z\rangle = S(x+y)\cdot Sz = (Sx + Sy)\cdot Sz =$

$Sx \cdot Sz + Sy \cdot Sz = \langle x,z\rangle + \langle y,z\rangle$

c. constant work

d. If $x \neq 0$ $\langle x,x\rangle = Sx \cdot Sx = \|Sx\|^2 > 0$ If $Sx \neq 0$,

$x \notin \ker(S)$ $Sx \neq 0$ when $x \neq 0$, $\ker(S) = \{0\}$

so S must be invertible (its a dot product)

b $\langle x,y\rangle = (Sx)^T Sy = x^T S^T S y$ So $S^T S = I$

S orthogonal

15 By defn $b = \left\langle \begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix} \right\rangle = \left\langle \begin{bmatrix}0\\1\end{bmatrix}, \begin{bmatrix}1\\0\end{bmatrix} \right\rangle = c$

so $b = c$. If $b=c$, $\langle v,w\rangle = \langle w,v\rangle$

b & c can be satisfied

Verify $\langle x,x\rangle > 0$

$= x_1^2 + 2b x_1 x_2 + d x_2^2 > 0$

$\hookrightarrow (x_1 + b x_2)^2 + (d - b^2) x_2^2 > 0$ iff

$d - b^2 > 0 \Rightarrow d > b^2$

so $b = c$ $d > b^2$

16a o nml basis

standard basis: $1, t$

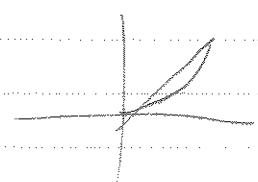
$\|1\| = \sqrt{\int_0^1 dt} = 1$ $g_2 = \dfrac{t - \langle 1,t\rangle 1}{\|t - \langle 1,t\rangle 1\|} = \dfrac{t-\frac{1}{2}}{\|t-\frac{1}{2}\|}$

$= \sqrt{3}\,(2t-1)$

b. $\mathrm{Proj}_{P_1}(t^2) = \langle 1, t^2\rangle 1 + \langle \sqrt{3}(2t-1), t^2\rangle \sqrt{3}(2t-1)$

$\langle 1, t^2\rangle = \frac{1}{3}$ $\langle \sqrt{3}(2t-1), t^2\rangle = \sqrt{3}\int_0^1 2t^3 - t^2 = \frac{\sqrt{3}}{6}$

$= t - \frac{1}{6}$



8.1 10, 14, 42

10. $\lambda_1 = \lambda_2 = 0 \qquad \lambda_3 = 9$

$v_1 = \frac{1}{\sqrt{5}}\begin{bmatrix}2\\1\\0\end{bmatrix}$ is in E_0, $\quad v_2 = \frac{1}{3}\begin{bmatrix}1\\-2\\2\end{bmatrix}$ in E_9

$v_3 = v_1 \times v_2 = \frac{1}{3\sqrt{5}}\begin{bmatrix}2\\-4\\-5\end{bmatrix}$ → o. normal eigenbasis

$$S = \begin{bmatrix} \frac{2}{\sqrt{5}} & \frac{1}{3} & \frac{2}{3\sqrt{5}} \\ \frac{1}{\sqrt{5}} & -\frac{2}{3} & -\frac{4}{3\sqrt{5}} \\ 0 & \frac{2}{3} & -\frac{\sqrt{5}}{3} \end{bmatrix} \qquad D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

14. $S^{-1}AS = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

a. $2A$, so $S^{-1}(2A)S = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 6 \end{bmatrix}$

b. $A - 3I_3$

$$S^{-1}(A - 3I_3)S = S^{-1}AS - 3I_3 = \begin{bmatrix} -3 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

c. $\frac{1}{2}(A - I_3) \qquad S^{-1}\left(\frac{1}{2}(A - I_3)\right)S = \frac{1}{2}\left(S^{-1}AS - I_3\right)$

$$= \begin{bmatrix} -\frac{1}{2} & 0 & 0 \\ 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

9.1 31. $\lambda_1 = 1$ $\lambda_2 = 6$ $\lambda_3 = 0$ $v_1 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$

since $x(0) = \vec{v}_1$, we don't need to Find v_2 & v_3

since $c_1 = 1$, $c_2 = c_3 = 0$.

so $x(t) = e^{t} \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$

52. eigenvalues of $A = \begin{bmatrix} \lambda & 1 \\ 0 & \lambda \end{bmatrix}$ are λ.

so $E_\lambda = \ker \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \rightarrow$

9.3 $p_t = \lambda^2 - 3\lambda - 10$ $(\lambda+5)(\lambda-2) = 0$

$x(t) = c_1 e^{-5t} + c_2 e^{2t}$ c_1, c_2 arbitrary

Genetics

$$M = \begin{bmatrix} 1 & \frac{1}{4} & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{4} & 1 \end{bmatrix} \qquad \text{eigenvalues} \quad \begin{aligned} \lambda_1 &= 1 \\ \lambda_2 &= 1 \\ \lambda_3 &= \tfrac{1}{2} \end{aligned}$$

$$v_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \qquad v_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad v_3 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

$$S = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & -2 \\ 0 & 1 & 1 \end{bmatrix} \qquad S^{-1} = \begin{bmatrix} 1 & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 1 \\ 0 & -\frac{1}{2} & 0 \end{bmatrix}$$

So $\quad x^{(n)} = M^n x^{(0)}$

$$= \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & -2 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & (\frac{1}{2})^n \end{bmatrix} \begin{bmatrix} 1 & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 1 \\ 0 & -\frac{1}{2} & 0 \end{bmatrix} \begin{bmatrix} a_0 \\ b_0 \\ c_0 \end{bmatrix}$$

$$a_n = a_0 + \left[\frac{1}{2} - \left(\frac{1}{2}\right)^{n+1}\right] b_0$$

$$b_n = \left(\frac{1}{2}\right)^n b_0$$

$$c_n = c_0 + \left[\frac{1}{2} - \left(\frac{1}{2}\right)^{n+1}\right] b_0$$

$\lim_{n\to\infty}$ $\quad a_n \to a_0 + \frac{1}{2} b_0 \qquad c_n \to c_0 + \frac{1}{2} b_0$

$b_n \to 0$

Game There

$$\begin{array}{ccc} & & \text{Strain} \\ \text{\#3 Pay off Matrix} & \text{Vaccine} & \begin{bmatrix} .85 & .70 \\ .60 & .90 \end{bmatrix} \end{array}$$

a